

# Reinforcing Math Skills in 7th-10th Grade

22. Satellite 1 makes a circular orbit around the Earth with a radius  $r_1 = R$ . Satellite 2 makes a circular orbit around the Earth with a radius  $r_2 = 2R$ . We let  $v$  represent the speed of a satellite and  $a$  represent the magnitude of a satellite's acceleration. Which one of the following choices gives the correct relation between the speeds and accelerations of the satellites?

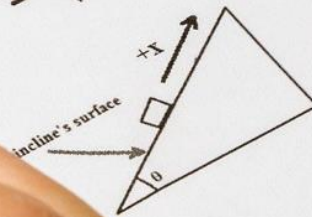
- (A)  $v_2 = \frac{1}{\sqrt{2}}v_1$  ;  $a_2 = \frac{1}{4}a_1$
- (B)  $v_2 = \frac{1}{2}v_1$  ;  $a_2 = \frac{1}{4}a_1$
- (C)  $v_2 = \frac{1}{\sqrt{2}}v_1$  ;  $a_2 = \frac{1}{2}a_1$

- (D)  $v_2 = \frac{1}{2}v_1$  ;  $a_2 = \frac{1}{2}a_1$
- (E)  $v_2 = v_1$  ;  $a_2 = \frac{1}{2}a_1$



23. A car moves with constant speed around a horseshoe-shaped path as shown with the arrows in the figure. Which one of the following choices best describes the direction of the average acceleration of the car in traveling from W to X?

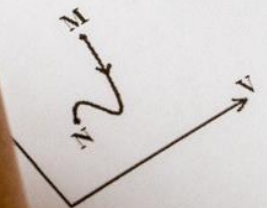
- (A) ↙
- (B) ↖
- (C) ↗
- (D) →
- (E) There is no average acceleration



24. A mass on a frictionless incline has a gravitational force  $F_g$  acting vertically downward. The mass remains at rest and the incline makes an angle  $\theta$  with the horizontal. Which one of the following choices best describes the orientation of the applied force by the incline's surface on the mass?

- (A) The applied force is parallel to the incline and points upward.
- (B) The applied force is perpendicular to the incline and points upward.
- (C) The applied force is perpendicular to the incline and points downward.
- (D) The applied force is parallel to the incline and points downward.
- (E) This is a completely impossible situation.

Which one of the following choices best describes the total force on the mass?



# ABOUT THIS RESOURCE

## Supporting school improvement and quality teaching

Our *Learners First Strategy* strengthens the quality of teaching and learning opportunities for all students in our system. In order to work effectively, well developed teaching and learning programs need to be implemented in every school, supported by strong, instructional leadership.

We know that it is good teaching that makes the difference to our students. The rich resources that are provided in the *Good Teaching* series are successfully supporting teachers and school leaders to continue to build both collaborative practice and a whole school approach to school improvement K–12.

Building on the *Good Teaching* series and aligned to the *Supporting Literacy and Numeracy Success* booklet, a set of literacy and numeracy resources has been developed to give teachers in the early years through to Year 12 more support and confidence when planning for students' literacy and numeracy needs across the curriculum. This particular resource focuses specifically on teaching numeracy 7–10.

## Supporting professional learning

Our *Learners First Strategy* aims to develop successful, skilled and innovative Tasmanians. Its values include learning and excellence so that Tasmanians are engaged in positive, productive and supported learning experiences, and have high expectations and a strong commitment to the pursuit of excellence.

As with the other *Good Teaching* resources, this resource will be accompanied by a professional learning program through the Professional Learning Institute (PLI) available to all schools.

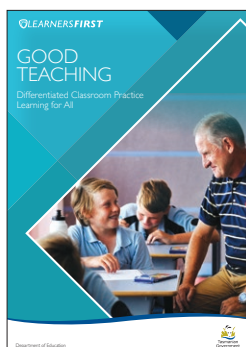
It forms part of the *Good Teaching* series of resources that also includes:



### ***Good Teaching: A Guide for Staff Discussion***

The purpose of this guide is to raise the debate across schools to gain a common understanding of what makes a good teacher. It is the foundation of the *Good Teaching* series.

[www.education.tas.gov.au/documentcentre/Documents/Good-Teaching-Guide.pdf](http://www.education.tas.gov.au/documentcentre/Documents/Good-Teaching-Guide.pdf)



### ***Good Teaching: Differentiated Classroom Practice – Learning for All***

It is recognised that some students require significant adjustments to their learning programs if they are to be optimally engaged and challenged. The process of making those adjustments is known as the differentiation of classroom learning. Differentiation is what is expected of good teachers. The focus of this resource is to describe what is meant by differentiation and to provide practical strategies and tools that can be used to create meaningful and engaging learning experiences for all students.

[www.education.tas.gov.au/documentcentre/Documents/Good-Teaching – Differentiated-Classroom-Practice-Learning-for-All.pdf](http://www.education.tas.gov.au/documentcentre/Documents/Good-Teaching – Differentiated-Classroom-Practice-Learning-for-All.pdf)

Practical examples are provided using the following identifiers:



Template



Good Practice



Video



Tool



Resources



Conversation Starters

## How the content is organised:

The booklet is divided into colour coded sections. Each section begins with key messages for Years 7–10 educators followed by conversation starters to initiate rich discussion in staff meetings or professional learning communities.

As the *Australian Curriculum: Mathematics* has a central role in the development of numeracy in a manner that is more explicit and foregrounded than is the case in other learning areas (ACARA, 2015) the intended audience for this resource is primarily teachers with responsibility for mathematics in Years 7–10. However, it also has important messages about the opportunities and demands for numeracy across the curriculum. It is therefore valuable reading for all staff members as schools consider whole school approaches to the development of numeracy as a capability.

In each of the sections A-F there is a focus on specific links to the elements of the general capability of numeracy and the *Australian Curriculum: Mathematics* to support classroom teachers in knowing the key teaching focuses at their year level. These sections also provide practical ideas for teachers and suggestions for the types of activities, questions and materials they might use to support student learning of numeracy.

At the back of the booklet there are references and recommended resources (including assessment tools) to provide additional support to teachers and school leaders for a more thorough appreciation of the key messages.

## This resource should also be used in conjunction with:

*Supporting Literacy and Numeracy Success* which provides teachers with strategies for improving literacy and numeracy outcomes as they plan using curriculum documents.

[www.education.tas.gov.au/documentcentre/Documents/Supporting-Literacy-and-Numeracy-Success.pdf](http://www.education.tas.gov.au/documentcentre/Documents/Supporting-Literacy-and-Numeracy-Success.pdf)

*Respectful Schools Respectful Behaviour* which highlights the importance of providing safe and supportive environments as a vital part of quality teaching and learning.

[www.education.tas.gov.au/documentcentre/Documents/Respectful-Schools-Respectful-Behaviour.pdf](http://www.education.tas.gov.au/documentcentre/Documents/Respectful-Schools-Respectful-Behaviour.pdf)

*Curriculum in Tasmanian Schools K–12 Policy*

[www.education.tas.gov.au/documentcentre/Documents/Curriculum-in-Tasmanian-Schools-K-12-Policy.pdf](http://www.education.tas.gov.au/documentcentre/Documents/Curriculum-in-Tasmanian-Schools-K-12-Policy.pdf)

*Assessment and Reporting Policy*

[www.education.tas.gov.au/documentcentre/Documents/Assessment-and-Reporting-Policy.pdf](http://www.education.tas.gov.au/documentcentre/Documents/Assessment-and-Reporting-Policy.pdf)

## NAPLAN Toolkit



The NAPLAN Toolkit supports teachers with strategies for teaching key concepts in literacy and numeracy. <http://naplan.education.tas.gov.au>

# BECOMING NUMERATE

## Numeracy across the years of schooling

Students become numerate as they develop the knowledge, skills and dispositions to use mathematics confidently across their years of schooling K–12, across all learning areas and in their lives more broadly. Students develop their knowledge, skills and confidence with numeracy as they connect and apply their understandings of mathematics to contexts outside the classroom.

In the *Australian Curriculum*, numeracy is a general capability, extrapolated in a numeracy learning continuum. The numeracy learning continuum of the *Australian Curriculum* presents a sequence of learning independent of student age, and is labelled from levels 1 to 6 indicating the particular level that typically applies to students by the end of a given year of schooling.

In the 7–10 years, numeracy is a key driver of learning across all *Australian Curriculum* areas. Students use their understandings of numbers, patterns, measurement, spatial reasoning and data across different learning areas. Teachers support students to analyse a situation to identify the mathematical ideas involved, asking the question: 'How can maths help here?' Students use mathematical skills and understandings for increasingly specialised purposes and audiences in a range of contexts. In doing so, students become confident communicators, critical thinkers, and informed young people who understand the world around them.

A commitment to numeracy development is an essential component of learning areas across the curriculum and a responsibility for all teachers.

## Numeracy across the curriculum

Students become numerate as they engage with numeracy opportunities and experiences across the learning areas of the *Australian Curriculum*. Numeracy is seen in action when students understand the role of mathematics and have the dispositions and capacities to use mathematical knowledge and skills purposefully. 'When teachers identify numeracy demands across the curriculum, students have opportunities to transfer their mathematical knowledge and skills to contexts outside the mathematics classroom. These opportunities help students recognise the interconnected nature of mathematical knowledge, other learning areas and the wider world, and encourage them to use their mathematical skills broadly' (ACARA, 2015).

Examples of becoming numerate in learning areas across the curriculum can be found throughout the *Australian Curriculum* website. The following examples are from the *Australian Curriculum* learning areas.



In **English** students develop numeracy capability when they interpret, analyse and create texts involving quantitative and spatial information such as percentages and statistics, numbers, measurements and directions.

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In **Mathematics** students develop numeracy in a manner that is more explicit and foregrounded than is the case in other learning areas. It is important that the Mathematics curriculum provides the opportunity to apply mathematical understanding and skills in context. A particularly important context for the application of Number and Algebra is financial mathematics. In Measurement and Geometry, there is an opportunity to apply understanding to design. The twenty-first century world is information driven, and through Statistics and Probability students can interpret data and make informed judgments about events involving chance.

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In **Science** students develop numeracy capability when they collect both qualitative and quantitative data, which is analysed and represented in graphical forms and through learning data analysis skills, including identifying trends and patterns from numerical data and graphs.

In later years, numeracy demands include the statistical analysis of data, including issues relating to accuracy, and linear mathematical relationships to calculate and predict values.

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### Humanities and Social Sciences (HASS)

- In **History** students develop numeracy capability as they learn to use scaled timelines, including those involving negative and positive numbers, as well as calendars and dates to recall information on topics of historical significance and to illustrate the passing of time.
- In **Geography**, students develop numeracy capability as they investigate concepts of location and distance, spatial distributions and the organisation and management of space within places; in constructing and interpreting maps, students work with numerical concepts of grids, scale, distance, area and projections.
- In **Economics and Business** (from Year 5) students use numeracy to understand the principles of financial management, and to make informed financial and business decisions.

- In **Civics and Citizenship** (from Year 3) numeracy knowledge and skills are applied to analyse, interpret and present information in numerical and graphical form, including conducting surveys and representing findings in graphs and charts.

Across **the Arts** subjects, students use spatial reasoning to solve problems involving space, patterns, symmetry, 2D and 3D shapes; scale and proportion and measurement to explore length, area, volume, capacity, time, mass and angles.

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In **Technologies** students cost and sequence when making products and managing projects. They use three-dimensional models, create accurate technical drawings, work with digital models and use computational thinking in decision-making processes when designing and creating best-fit solutions.

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In **Health and Physical Education** students use calculation, estimation and measurement to collect and make sense of information related to nutrition, fitness, navigation in the outdoors or various skill performances. They use spatial reasoning in movement activities and in developing concepts and strategies for individual and team sports or recreational pursuits.

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In **Languages** there are opportunities for learners to use the target language to develop skills in numeracy, including processes such as using and understanding patterns, order and relationships to reinforce concepts such as number, time or space in their own and in others' cultural and linguistic systems.

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In **Work Studies** (Years 9–10) students strengthen their numeracy skills by making direct connections between their mathematical learning and the nature of mathematics required in workplaces and enterprises. Students recognise that financial literacy is a requirement across enterprises and that numeracy helps them manage salaries and personal and workplace budgets and calculate personal and enterprise tax liabilities.

# WHOLE SCHOOL APPROACHES TO NUMERACY

*Numeracy is regarded as one of the key dimensions of all learning. Growth in numeracy outcomes will only occur when there is a sustained whole school commitment to systematic curriculum delivery over a period of time. Improvement in numeracy achievement requires a whole school commitment to the following key aspects:*

## Organisation

A culture of collaboration empowers staff to work together on numeracy provision, discussing, reflecting, planning, setting goals, developing resources, analysing data and work samples, and sharing learning. An effective numeracy improvement strategy benefits from leadership by a numeracy leader and/or a numeracy team in managing and leading:

- planning and review cycles
- collecting and analysing data
- target setting
- ensuring coherence and continuity across the years
- promoting formative assessment strategies to guide numeracy teaching
- building staff capacity and confidence
- enabling the sharing of effective numeracy practices across the school
- supporting the principal to develop whole-of-school interventions for students requiring differentiated numeracy support
- identifying resource needs and allocation
- convening and structuring year group numeracy planning meetings
- ensuring ongoing moderation of student work to build consistency of judgements and teachers' understanding of expected curriculum outcomes.

## Planning

It is important for schools to allocate time for year level meetings and to develop structures to support ongoing collaboration so that teachers can plan how to teach mathematics well and identify where mathematics applies to other areas of the curriculum. Teachers collaboratively decide on consistent approaches to teaching practices, assessment and task selection. They consider how the vocabulary of mathematics/ numeracy will be explicitly modelled and taught and which models will be used to support learning. Together, teachers develop and organise shared resources, including online materials.

Principals and curriculum leaders have a key role in supporting teachers to understand the *Australian Curriculum: Mathematics* – the individual content descriptors and the expected outcomes as described in the achievement standards for each year group. They encourage teachers to refer to the glossary for mathematics and the work samples which are published online. Teachers also need to understand how to design and use tasks which build students' mathematical proficiencies of fluency, understanding, problem solving and reasoning, and their capacity to apply mathematics in a range of curriculum contexts.

Teacher planning includes differentiating the curriculum for students requiring additional numeracy support, including students who are not making expected progress. Refer to *Good Teaching: Differentiated Classroom Practice and Supporting Literacy and Numeracy Success*.

Because secondary mathematics teachers are more likely to have specialist discipline knowledge, they understand the stages in the *Australian Curriculum* that students need to progress through and can intervene as necessary, or advise other learning area teachers. Teachers should always believe that appropriate intervention can assist students to learn, often through some creative thinking and thoughtful, collaborative planning.

If some students find it difficult to keep up with maths learning, teachers need to group them according to their needs and provide targeted support while others in the class consolidate more recent learning. Differentiation strategies are described in *Good Teaching: Differentiated Classroom Practice*. Usually this will mean continuing to teach the whole class new material

but then working with individuals and small groups as time permits during lessons. Many schools use other strategies such as 'catch-up classes' during lunch and after school, or before-school tutorials. It is essential to attend to the learning needs of struggling students because a sense of failure can be linked to behavioural issues and disengagement. And, of course, these students will continue to fall further behind unless they are adequately supported.

Because the expectations of the *Australian Curriculum* are described in terms of what students should be able to *know, do and understand*, teachers need to visualise what students would be doing and how they would be acting if they could do what is specified. Teachers then need to ask: *What do I need to do to enable them to do and know these things?* This is the pedagogy question.

## Teaching and learning

**An inquiry approach to teaching and learning numeracy is recommended for all years of schooling.** This involves a focus on problem solving and opportunities for students to apply mathematical skills purposefully across the curriculum.

The mathematics that underpins numerate behaviours is first learned as a body of knowledge in the primary years of schooling. By understanding a context and determining that 'some mathematics will help here', students then make some choices about what mathematics will help and what strategies they will use to apply the mathematics selected.

Numeracy requires problem-solving skills in order to understand context and make choices about strategies. Problem solving is one of the four Mathematical Proficiencies<sup>i</sup> that should be used in teaching mathematics. However, the key numeracy question – *will some Maths help here?* – is usually asked outside of, or prior to, mathematics lessons. Hence it is part of *general* problem solving which can be used in every context (including other learning areas) and not just in the teaching and learning of mathematics.

A recommended problem-solving framework<sup>ii</sup> has the following steps, not always applied consecutively:

1. **Clarify** the problem.
2. **Choose** mathematics, tools, procedures and skills to solve the problem based on the clarification.
3. **Use** and/or apply what has been chosen.
4. **Interpret** and check appropriateness of application and solution.
5. **Communicate** all steps in finding the solution for appropriate purpose and audience.

Students need to be explicitly taught how to undertake these steps through a consistent whole school approach if they are to be numerate. Knowing mathematics is essential but it is not sufficient. Students need literacy skills to help 'read' and comprehend contexts; confidence in their mathematics knowledge and skills to choose those that 'fit' the context; connect them; use them and apply them; and higher-order skills to determine whether their answers make sense in the context – and then to go back and clarify and re-choose if they don't.

Students can then apply the mathematics and strategies chosen confidently and make a judgement about whether their solution makes sense in the particular context. If their solution does make sense then they gain confidence in their application of mathematics and are more likely to choose and use mathematics next time they identify it as being needed. Continued success will mean students become more and more numerate in many different contexts.

To be confident problem solvers, students need to develop favourable attitudes towards applying their mathematical knowledge and be able to make strategic choices about which strategies and concepts they will draw on in which contexts. This requires careful planning for tasks which promote and demand problem solving, thinking and inquiry along with reasoning and communication of solutions and strategies. Refer to *Good Teaching: Differentiated Classroom Practice and Supporting Literacy and Numeracy Success*.

The emphasis in Years 7–10 is on supporting students to continue to deepen their understanding of mathematics, to learn to use it across the curriculum when required, and to apply mathematics to a range of situations beyond the classroom. All teachers support this learning and contribute to the development of students' numeracy.



## Effective numeracy teachers:

Effective Years 7–10 teachers recognise that they have particular roles in the development of students' numeracy:

1. Know their students: What cultural backgrounds do students come from? What language backgrounds do they have? What are their individual learning needs? Teachers should talk to students about how they learn and their preferences.
2. Know what knowledge and understandings of mathematics Years 7–10 students bring with them to the learning context. They use systems such as *edj* to understand students' educational history and past assessment data, including NAPLAN results. This can support transition from primary schooling into Year 7 and can also support transitions from Year 10 to Year 11.
3. Know the expected learning outcomes for mathematics and numeracy for students in this age range, including knowing the expectations before and after the years they are teaching and how the concepts in mathematics develop.
4. Know what mathematics and problem-solving skills students may have learned in their previous years of schooling. They ask: Have they learned what they are expected to learn? Have they been taught the mathematics of the Prep (Foundation) – Year 6 *Australian Curriculum*? Have they acquired the problem-solving skills they need to choose and apply mathematics appropriately in contexts that demand it?

**Teachers of mathematics** working with students in Years 7–10 then need to:

- Teach the mathematics needed to undertake numeracy well. **Teaching mathematics well** implies the need to teach for understanding of mathematics **concepts**, rather than teaching **content** and the **methods** and **procedures** needed to deal with the content. Teachers need to fully understand the mathematics concepts, before they can effectively teach them to students. This resource booklet does not simply 're-package' mathematics content. It explains the concepts and deep understandings that students need to be truly 'numerate' and how to teach **mathematics for numeracy**.
- Read the K–2 and 3–6 resource booklets in this series to find out exactly what deep understandings students should have in order to access what will be taught in Years 7–10.
- Revise what students should have learned earlier, give them tasks to demonstrate this learning, and then move around groups, identifying gaps and/or misunderstandings in students' concept and skill base.
- Re-teach concepts and skills to individual students as required. To be able to do this, teachers need a sound understanding of the sequential 'building blocks' of the mathematics curriculum, so they can take students back to an appropriate starting point from which to scaffold their new learning. The Sequence of Content charts in the *Australian Curriculum* offer teachers a guide to where students should be in their learning and to the essential learning that precedes each stage of understanding.
- Work from the premise that all students can learn and will do so with appropriate teaching and support, including an emphasis on building a growth mindset.

### Teachers of subjects other than

**mathematics** working with this age group need to have a broad knowledge of mathematics to understand the relevance of that content to their own specialist subject areas. They need to know when mathematics is needed and how it might be applied.

Their role is to:

- Ask students: 'Will some mathematics help here?'
- Point out to students that: 'We are using some mathematics here' to indicate to students that 'mathematics is in everything; it's not just a stand-alone subject that we do at school'.
- To 'speak aloud' their decision-making and processes when they apply their mathematics. For example, in Science, they might say: 'We will now put our results in a table (on a chart). Will some maths help here? What sort of table (chart) will work best to show our results? Let's think about the audience; who will need to interpret this data?' Modelling the use of mathematics to your students *during the teaching of your own subject area* is essential in order for students to learn the broad application of mathematics to contexts outside of their mathematics lessons.

Learning areas other than mathematics require applications of mathematics that are *subject specific* ... they have been adapted and have evolved over time to work best in specific contexts. For example, in a Design and Technology class, teachers are aware that most measurement of timber is done in millimetres. They are also aware that all measurements need to be accurate since mistakes can be inefficient and costly. Therefore they might demonstrate measurement in different ways than those demonstrated and learned in mathematics lessons. Teachers of this learning area should discuss these differences with the mathematics teachers in the school for two reasons:

- So that teachers can explain to students the specific numeracy demands of particular subjects and how maths terminology and procedures may differ from the maths classroom to a learning area such as Technologies.

- So that students comprehend that the key understandings that they are learning in maths have wide application from context to context but that these are adapted for different learning areas and contexts; the differences depend on **audience and purpose**.

It is not appropriate to attempt to provide information for teachers about all the possible contexts in Years 7–10 where different numeracies are needed or applied. However, it is clear that, to contribute to a whole school numeracy approach, teachers of all subjects need to be informed about how and when mathematics is relevant to their learning area. They should ask the mathematics teachers in the school to help them identify such situations if they are unsure. Be aware that your students might be numerate in some subjects but not your own.

## Support for teachers

- Teachers have varying levels of experience and expertise in different aspects of numeracy education and they therefore require different levels of professional support.
- Having a numeracy leader or a numeracy team who, along with senior staff, can work with teachers 'shoulder to shoulder' as well as identifying their professional learning needs, underpins school improvement in this area of the curriculum.
- Professional learning is more effective when it is student-focused, data-informed and sustained, rather than in one-off sessions.
- Teachers need opportunities to learn about the mathematical ideas that underpin numeracy and the evidence-based teaching approaches that support student learning.
- Teachers need to be able to articulate both what they do and why they do it.

When these professional supports are in place, and when effective numeracy practices are shared across the school, whole school improvement is sustained.



## Assessment

To ensure continuity of numeracy development, it is important to develop a consistent approach to assessment, as outlined in *Good Teaching: Quality Assessment Practices*.

- Whole school practices for collation of data and reporting procedures support planning and tracking of student achievement.
- Teachers and leaders work together to investigate patterns of students' strengths or underachievement and plan for interventions based on information from the data. Refer to *Supporting Literacy and Numeracy Success*.
- Assessment should lead to more effective teaching with teams developing a plan of action and selecting focus areas for improvement.
- Progress is monitored and teaching is adjusted accordingly.
- Year group teachers benefit from sharing formative numeracy assessment practices and planning for adjustments to teaching as a result of new understandings of learners.
- Success criteria should be shared with students who increasingly take responsibility for addressing the criteria and assessing their own numeracy progress.

# NUMERACY 7–10

## The numeracy learning environment

*This resource identifies the key aspects of mathematics students in Years 7–10 need to understand in order to become numerate as they engage with numeracy across the curriculum and increasingly in their daily lives. It describes prerequisite features for numeracy learning as well as explicating the key elements of numeracy*

A classroom setting that encourages numeracy learning includes:

- Learning spaces designed to facilitate whole class, group, pair and individual work.
- Numeracy materials organised for independent learning e.g. counters, dice, measuring tools, calculators.
- Opportunities for experiential hands-on learning and classroom discussions focused on explaining and sharing of learning.
- Various technologies e.g. individual mini-whiteboards, tablets, laptops, visualisers to project student work to support class discussion and sharing of thinking.
- Teacher/student made materials and posters (rather than commercial materials) that have a meaningful connection to the curriculum and are effective tools for teaching and learning.
- Frequently referenced materials e.g. mathematics dictionaries, number charts, number lines.
- Displays that are fresh, uncluttered and purposeful.
- Student work on display that shows thinking as well as their conclusions.



## Planning and teaching for numeracy

In Years 7–10 teachers connect numeracy learning to the Numeracy general capability of the *Australian Curriculum*, underpinned by Years 7–10 of the *Australian Curriculum: Mathematics*. Effective planning and teaching emphasises backward design and the importance of clear links between learning goals and assessment tasks. Refer to *Good Teaching: Curriculum Mapping and Planning*. It includes:

- Assessing student understandings about concepts through oral questioning, watching students go about problem-solving and inviting input from parents about how their children deal with numeracy at home.
- Determining the mathematics that needs to be learnt by backward mapping from curriculum outcomes (both achievement standards and content descriptors from the *Australian Curriculum: Mathematics* and/or the expected indicators in the Numeracy Learning continuum).
- Determining the scaffolding and tasks needed to bridge between prior understandings and new numeracy and mathematical ideas.
- Determining contexts to teach the numeracy in familiar and engaging ways that promote discussion and focus on problem-solving.
- Facilitating inquiry learning, with planned opportunities for explicit focus on the mathematical ideas needed for numeracy.
- Teaching numeracy and mathematics in ways that promote understanding, support fluency, and demand reasoning and problem-solving.
- Determining the tasks required for students to demonstrate their understanding of numeracy.
- Generating data for diagnosing future learning and intervention needs, such as diagnostic assessment, benchmarking, outcomes assessment, success criteria (including rubrics), observation checklists, portfolios of student work and parent interviews (see Appendix).



## Monitoring and assessment

In order to monitor students' learning against what they are expected to learn, teachers need to refer back to the descriptors and standards in the *Australian Curriculum*. Teachers need to be very clear about what their goals are for these particular students, to share those goals with students and to review progress in an ongoing way. Ongoing formative assessment, including pre-assessment helps teachers know where students are in their learning and informs planning. Refer to *Good Teaching: Quality Assessment Practices*.

Assessment must focus on assessing the intended learning as opposed to what is merely taught. Assessing whether students *deeply* understand the numbers, operations and strategies you have taught them requires you to assess whether students can make choices about which of these to use to solve problems. On the other hand, if you have spent a week teaching addition of double-digit numbers and then assess this skill and understanding, students will only be demonstrating that they have acquired what has been recently taught, not that they can use the knowledge in new contexts.

'Testing what you've just taught' may be appropriate for Mathematics teaching and learning, but is not valid for the assessment of the numeracy capability. To assess **numeracy** teachers need to assess whether their students can choose which mathematics is needed in the context of each question. To arrive at a sensible and plausible result students need to bring to the task:

- a deep knowledge of the mathematics that will maximise their appropriate choices
- a facility with the strategies and methods that will maximise their ability to choose these and calculate correctly
- an ability to reason mathematically about whether their obtained solutions make sense in the given, understood, context.

To assess the deep understandings of mathematics, as described above, teachers must ensure that they have taught the deep understandings to all students using a high expectations approach and differentiation strategies as described in *Good Teaching: Differentiated Classroom Practice*. This may include teachers providing extending or enabling prompts for tasks/problems to broaden student access to the learning.

Monitoring this mathematics learning should be done by determining – at the point in time of this assessment task – the extent to which students know and understand a specific content descriptor and what support they then need in order to move to the next cognitive level. For example, if they can only *recall*, then they need targeted teaching to help them *understand* the concepts learned. If they are unable to understand a context sufficiently well to choose the appropriate operation from the words given and explain this choice, then they need targeted support to visualise, to choose more strategically, and to explain their choices.

If students don't appear to be progressing it is important to have an objective look at your assessment tasks:

- Are the questions too hard?
- Do they assess the learning that you planned for?
- Are there words and phrases in the tasks that students may not be able to read and understand?
- Do your observations validate the learning, or are you seeing different things in the behaviours than you are seeing in written tasks?

Have a peer or mentor check the alignment between the tasks and the intended learning.



### *For leaders:*

How is numeracy reflected in the School Improvement Plan? How will we monitor progress toward our goals?

What organisational provisions are in place for developing whole school numeracy across the curriculum? e.g. instructional leadership, collaborative planning teams, appropriate resourcing and provision of professional learning

How are these responsibilities for numeracy distributed e.g. is there a numeracy leader or a numeracy team?

How successful is the collaborative planning for numeracy teaching and assessment in the school?

What are some common numeracy assessments the school uses?

How are the diverse numeracy needs of students catered for?

How is numeracy communicated to parents and carers?

How are numeracy interventions managed and resourced?

How is data collection and analysis of numeracy managed in the school?

### *For teachers:*

What types of assessments provide you with a range of numeracy data about your students?

What learning experiences assist your students to develop numeracy competence and confidence? What has worked well? What could be improved?

In what ways is the learning environment conducive to numeracy learning? What could be improved?

How do you design learning tasks that allow students to develop and demonstrate numeracy understanding?

How is numeracy learning differentiated to meet the needs of all students?

How is numeracy evident in planning across the curriculum?

Are there areas of numeracy education in relation to which you feel you might benefit from professional learning?

# NUMERACY KEY ELEMENTS 7–10

The organising elements for Numeracy are:

- Estimating and calculating with whole numbers
- Recognising and using patterns and relationships
- Using fractions, decimals, percentages, ratios and rates
- Using spatial reasoning
- Interpreting statistical information
- Using measurement.

Note the higher-order cognitive functions captured in the verbs used; words such as *estimate* and *interpret* are at a higher level than others used in the *Australian Curriculum: Mathematics* content descriptors such as *recall*, *investigate*, *count* and *develop*. The words in the numeracy elements reinforce to teachers the **depth of learning of mathematics students need** in order to choose to apply mathematics to contexts outside the mathematics classroom. Students who gain mathematics knowledge in a superficial way – lacking deep understandings – are unlikely to have the confidence to choose to use mathematics when they can choose not to. They are unlikely to become fully numerate or to demonstrate sophisticated numerate behaviours.

The advice that follows concerning both the teaching and assessment of Numeracy key elements 7–10 is aimed at developing students' deep understandings. It therefore describes appropriate pedagogies and assessment tasks/questions that develop and elicit higher-order responses from students.

## A. Estimating and calculating with whole numbers



### Key messages

The positioning of the words 'estimating' with 'calculating' at the front of this element is a key message for teachers: for numerate behaviour, all calculation requires an element of estimation at the outset. In order to determine that a solution makes sense in context – an important part of numeracy – we need to first have a sense of the nature of the solution we are expecting.

Estimation in calculation relies on **deep understandings about numbers and how they work** as well as deep understandings about mathematical operations. Neither of these understandings is necessarily required in standard written mathematics algorithms and computations. Teachers also need to know that teaching algorithms to students as a means of fostering fluency *before* deep understandings are established can have harmful effects<sup>iii</sup>. Students rarely deepen their understandings by only practising algorithms.

The key to teaching students how to estimate and calculate with whole numbers is to ensure that students *deeply*:

- Understand what whole and decimal numbers are and that they can be represented in words, numerals and objects/drawings, and on number lines
- Understand that whole and decimal numbers can be used in different ways – for quantity, to order and as labels
- Understand operations and how they can be used to represent and solve problems
- Understand how to break down and partition whole and decimal numbers to help solve problems.



Students should have learned these concepts for whole numbers in Years K–2. In Years 3–6 they continued to consolidate these concepts using larger numbers, including decimals. By the end of Year 6 students are expected to be able to do these things with all whole numbers. This progression, however, cannot be assumed and differences in student attainment should be considered in planning.

## Links to the curriculum

Links to the *Australian Curriculum: Mathematics* are with the Number and Algebra strand, Year 7 to Year 10, and to the numeracy learning continuum levels 5 and 6: Estimating and calculating with whole numbers.

Relevant Australian Curriculum: Mathematics Content Descriptors			
Year 7	Year 8	Year 9	Year 10
<p><b>Understand, represent and order numbers</b></p> <p>Investigate index notation and represent whole numbers as products of powers of prime numbers</p> <p>Investigate and use square roots of perfect square numbers</p>	<p>Use index notation with numbers to establish the index laws with positive integral indices and the zero index</p>		
<p><b>Operate with numbers</b></p> <p>Apply the associative, commutative and distributive laws to aid mental and written computation</p> <p>Compare, order, add and subtract integers</p>	<p>Carry out the four operations with integers, using efficient mental and written strategies and appropriate digital and written technologies</p>		
<p><b>Money and financial mathematics</b></p> <p>Identify and justify 'best buys' with and without digital technologies</p>	<p>Solve problems involving profit and loss, with and without digital technologies</p>	<p>Solve problems involving simple interest</p>	<p>Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies</p>

Relevant Australian Curriculum: Mathematics Content Descriptors			
Year 7	Year 8	Year 9	Year 10
Extracts from Australian Curriculum: Mathematics Achievement Standards			
<p>Solve problems involving the comparison, addition and subtraction of integers</p> <p>Make the connections between whole numbers and index notation and the relationship between perfect squares and square roots</p> <p>Solve problems involving percentages and all four operations with fractions and decimals</p> <p>Use fractions, decimals and percentages, and their equivalences</p> <p>They compare the cost of items to make financial decisions</p>	<p>Students use efficient mental and written strategies to carry out the four operations with integers</p> <p>They recognise index laws and apply them to whole numbers</p> <p>They describe rational and irrational numbers</p> <p>They solve problems involving profit and loss</p>	<p>Students solve problems involving simple interest. Students apply the index laws to numbers and express numbers in scientific notation</p>	<p>Students recognise the connection between simple and compound interest</p>

Australian Curriculum Numeracy Learning Continuum	
Level 5	Level 6
<p><b>Understand and use numbers in context</b> Compare, order and use positive and negative numbers to solve everyday problems</p>	<p>Use different ways to represent very large and very small numbers including scientific notation</p>
<p><b>Estimate and calculate</b> Solve complex problems by estimating and calculating using efficient mental, written and digital strategies</p>	<p>Solve and model problems involving complex data by estimating and calculating using a variety of efficient mental, written and digital strategies</p>
<p><b>Use money</b> Identify and justify 'best value for money' decisions</p>	<p>Evaluate financial plans to support specific financial goals</p>

There are certain mathematics topics that the everyday person does not need to know or be able to use in order to meet the demands of life and further learning. For example, the content descriptor *Investigate and use square roots of perfect square numbers* in the Australian Curriculum: Mathematics for Year 7 above, does not appear in Numeracy levels 5 or 6.

This is because that knowledge is specific to mathematics. The numeracy entries above focus primarily on the mathematics needed for broad application by the general public. The mathematics content descriptors that remain are part of mathematics learning but not essential for numeracy.

## Planning

Teachers use the information in the 'Links to the curriculum' table in order to plan. In particular, they need to understand precisely what the intended mathematics learning captured in the numeracy continuum, means. Focusing on the verbs is helpful. The verbs capture what students need to *know, do and understand* with respect to this element by the end of a particular year of the *Australian Curriculum* or *level in the Numeracy continuum*. For example, students should be able to *Solve problems involving profit and loss, with and without digital technologies* by the end of Year 8 if they are to achieve what is expected in Mathematics; and *Identify and justify 'best value for money' decisions* by the end of Year 8 if they are to be numerate.

The activities in the following 'Putting it into practice' section describe the sorts of learning opportunities teachers need to create to develop the required, stated learning, both for numeracy and the mathematics that underpins it.



### Putting it into practice

#### I. Compare, order and use positive and negative numbers to solve everyday problems (level 5); and Use different ways to represent very large and very small numbers including scientific notation (level 6)

These entries are part of the 'understand and use numbers in context' element as opposed to the 'estimate and calculate' element of the numeracy continuum. 'Compare, order and use' and 'represent' suggest understanding the numbers and using them to solve problems when calculation is not required.

Students were taught in primary school that numbers can be represented in different forms. They learned to model and represent numbers using words, numerals, objects, drawings, symbols and number lines. They also learned how to use these various forms to represent all *positive* numbers (i.e. numbers greater than zero) with increasing magnitude into the millions and decreasing to beyond hundredths. In Year 6 they learned to locate positive and negative whole numbers and zero on a number line.

From Year 7, they should now be able to represent and order any whole and fractional/decimal numbers on a number line.

(Note that 'positive and negative whole numbers and zero' are defined as **integers**; seven words are replaced by one). On a number line they look like the following:



Students have learned that negative numbers enable us to perform calculations such as  $4 - 7 = ?$ . Negative numbers enable us to calculate and represent negative temperatures (temperatures below  $0^\circ$ ), debts (amounts of money owing when you don't have any money) and floor levels that are underground in buildings. Note that these are all important, real-life contexts for solving problems. Although underground floor levels in buildings are represented by integers (make sure students see examples of vertical number lines not just horizontal ones), the other two examples describe continuous quantities, where not just integers are possible but also the fractional/decimal numbers in between. So by Year 7 students are working with *all* positive and negative numbers.



#### Some activities for place value learning

- Call out the following numbers and have students represent them on the same number line:
  - 35, 2.4, -5, -2.6, 4.75, 6.35, 12.01.They have to work in pairs to draw their number line, deciding where to put the zero, how long it needs to be to allow for the accuracy of two decimal places, and so on. You could do this task every day or every now and then just to ensure all students are thinking about positive and negative numbers and how to locate them on a number line. This will enhance their visual capability; they need to think *distance from zero* rather than *left to right*, which they have become familiar with when using number lines in the past.

- Have students write a number presented in words, using numerals: four hundred and two thousands, and seven. (Have students write a question like this each, put them all on the board and ask all students to write the answers.)
- Give students a number line with a group of numbers – such as those in the previous question – on a number line. Have them write the numbers as they would say them:

– 5	<i>Negative five</i>
– 2.6	<i>Negative two point six</i>
4.75	<i>Four point seven five</i>
12.01	<i>Twelve point zero one</i>

- Have students work in pairs to read, clarify and interpret the following problem:
  - I am in the lift of a tall building. The building has fifteen floors, two of which are below ground level. If I start on the third floor where I live, get in the lift, go up eight floors, down six, up two and then down to the basement (one floor below ground level) coming back to ground level to disembark, how many floors have I travelled? How many up and how many down? Students should draw a diagram of their journey and include the total number of floors they travelled upwards, and the total number they travelled downward.
- Have students work in pairs to write a similar problem to the one above and give to another pair of students to solve.

- Below is a table showing the heights of five very tall buildings:

<b>Building</b>	<b>Height (m)</b>
Omega Heights	454
Zephert Tower	322
18 Mills Avenue Apartments	389
NBN Tower	413
Sunshine Hotel	264

- How much taller than Zephert Tower is the NBN Tower?
- The Sunshine Hotel is how many metres shorter than Omega Heights?
- What is the difference in height between the two shortest buildings?
- What is the difference in height between the shortest and tallest buildings?
- Put the buildings in order of their height on a vertical number line.
- Use a greater than ( $>$ ) and a less than ( $<$ ) symbols to express the relationships between each of the buildings, in terms of their height.
- Students can be given question such as:
  - In 2011 there were 301 617 babies born and 154 996 were boys. How many of the babies were girls?
  - Two of Australia's biggest dams have capacities of 3.4 million cubic metres and 467 396 cubic metres. What is the difference in their capacities? Which dam holds the most and by how much?
  - The temperature in Alaska is known to vary by a large amount each day. On one day in December in a certain town the temperature dropped by  $28^{\circ}\text{C}$ . The maximum on that day was  $5^{\circ}\text{C}$ . What was the minimum? What was the temperature range on that day?

- In Tanzania, annual births are 1.9 million while the annual births of Spain are 483 thousand. How many more births per year are there in Tanzania than in Spain?
- Three girls were playing a game in which the winner earned eight points per round and the loser lost eight points per round. Diane finished the game with 42 points, Gemma with 240 points and Zoe with – 32 points. What was the difference between each of the girls' number of points?
- Which of these numbers is closest to 0: 2.002; 0.0002; – 0.02; – 0.022

**Representing very large and very small numbers**

can become tedious, but if many of the numerals have face values of zero we sometimes don't need to write all of them down. There is a simpler way of writing 6 700 000 for example: we write it as 6.7 million. Have students work in pairs to consider a shorthand way of writing 3 450 000, 25 000 000, 304 000 000, 56 709 985 203 and 650 000.

We can write all numbers using 'shorthand' ways by thinking of them as products of prime numbers. A **prime number** is a number that has only itself and one as factors. Prime numbers include 2, 3, 5, 7, 11 and 13 since the only numbers that divide into them evenly (with no

remainder) are themselves and one:  $2 = 2 \times 1$ ;  $3 = 3 \times 1$ ;  $5 = 5 \times 1$ ;  $7 = 7 \times 1$ ;  $11 = 11 \times 1$ ; and  $13 = 13 \times 1$ . Have your students work in pairs to find out what the next five prime numbers are.

The next step is to write every other number as a product of prime numbers. For example,  $4 = 2 \times 2 \times 1$ ;  $6 = 3 \times 2 \times 1$ ;  $8 = 2 \times 2 \times 2 \times 1$ ;  $9 = 3 \times 3 \times 1$ , and so on. Have students work in pairs to find the *prime factors* of 20, 15, 27, 28, and 32.

You might think: this certainly doesn't look like a 'shorthand' way of writing numbers – in fact they all look longer! Although we have expanded the numbers initially, when we look at the prime factors we see that, in general, many numbers are made up of more and more 2s, 3s and 5s, we can see that they can be written in a shorthand way as follows:

$8 = 2 \times 2 \times 2 = 2^3$  (the superscript 3 indicates that 2 is multiplied by itself 3 times)

$9 = 3 \times 3 = 3^2$  (the superscript 2 indicates that 3 is multiplied by itself twice)

In general, any number (a) multiplied by itself three times is  $a^3$  or  $a \times a \times a$ .

This shorthand method involves new names for numbers;  $2^3$  is another way of writing 8, where '2' is the base and 3 is the index (or *power*). Numbers written with a base and an index are written in index form.

Have students work in pairs to complete the following table (add additional entries of your own):

Number	Prime Factors Expanded	Using Indexes (Indices)	Base(s)	Index(es)
9	$3 \times 3$	$3^2$	3	2
16	$2 \times 2 \times 2 \times 2$	$2^4$	2	4
27	$3 \times 3 \times 3$	$3^3$	3	3
12	$2 \times 2 \times 3$	$2^2 \times 3$	2, 3	2, 1
20	$2 \times 2 \times 5$	$2^2 \times 5$	2, 5	2, 1
60	$5 \times 2 \times 2 \times 3$	$5 \times 2^2 \times 3$	5, 2, 3	1, 2, 1



- Questions about indices:
  - Expand the following:  $2^3$ ;  $4^3$ ;  $x^2$ ;  $a^4$ ;  $3^2 + 3^2$
  - Write each of these using index notation:  $2 \times 2 \times 2$ ;  $z \times z \times z \times z$ ;  $3 \times 5 \times 3 \times 5$ ;  $m \times m \times 4$
  - Simplify:  $2^3 + 3^2$ ;  $5^3 + 3^4 + \frac{1}{2}$ ;  $7^2 - 3^3$ .

When students have learned about indices they are ready to learn about **scientific notation**. The system for writing most numbers is called **standard form**. For example, we can use scientific notation to write 123 000 as  $1.23 \times 10^5$ .

**To write a number using scientific notation** you write it as a single whole numeral followed by the decimal point; any other numerals become decimal numerals. You then multiply this number by 10 raised to the power of how many places you have made it smaller by in order to write it in this format.

Write the following large numbers using scientific notation i.e. in standard form:

123 000; 34 million; 295 600; 579 million; 247 982; 23 456 127.

#### Some further activities

- Write the following numbers using scientific notation: 0.0034; 0.0578; 0.0006; 0.000579
- Which of these numbers is closest to 1? 1.002; 1.1;  $1.11 \times 10^{-1}$ ; 0.111. Explain your answer
- Have students work together in pairs to determine which of the following numbers will have positive powers of ten when written in scientific notation and which will have negative powers of ten: 45.6; 0.045; 34.005; 0.37102; 0.0089; 3 million;  $\frac{1}{3000}$ ;  $\frac{1}{2}$ ;  $65.7 \times 10^3$ ;  $35.7 \times 10^{-2}$
- Have students work together to compare and order the following numbers, using '<' symbols and '>' symbols: 0.005;  $\frac{1}{5}$ ; 36 million;  $35.7 \times 10^3$ ; 0.0034;  $45.32 \times 10^{-2}$ ;  $\frac{1}{10000}$ ;  $2^3$ . See if they can do it without writing the numbers in standard form first (this will determine whether they really understand the number forms).

Consider the distances from the Sun of each of the following planets:

Planet	Distance from Sun (km)	Distance using Scientific Notation (km)
Mercury	57 910 000	
Venus	108 200 000	
Earth	149 600 000	
Mars	227 940 000	

The radius of a hydrogen atom is  $2.5 \times 10^{-11}$  m. What would be the diameter of a hydrogen atom?

## 2. Solve and model problems (level 6: involving complex data) estimating and calculating using efficient mental, written and digital strategies.

The greatest challenge for many students in solving word problems is that they don't understand the language/words used. All problems are written using words; teachers who try to take out the words or make them simpler so students can access them, do their students a grave injustice.

To be numerate students need to understand the words and to draw out the mathematics from them so that they can represent the situations described in words using symbols that facilitate calculation. Much has been written about this in the earlier K–2 and 3–6 resource booklets. However, you will not be surprised to find that many of your Year 7–10 students continue to struggle with this skill and you might wish to consider these excerpts and the pages they are drawn from, in greater detail.

### Teaching mental strategies

In order to estimate calculations students need to not only understand the magnitude of numbers they are working with but also deeply understand operations. In particular, they need to understand how each of the four operations (addition, subtraction, multiplication and division) relate to each other. While these are taught earlier, many students fail to consolidate them and they need to be re-taught and practised in ways that develop fluency.

Relationships include:

- Commutativity of multiplication and addition  
e.g.  $6 \times 3 = 3 \times 6$
- Inverse relationship between multiplication and division e.g.  $3 \times 4 = 12$  and  $12 \div 3 = 4$
- Inverse relationship between addition and subtraction e.g.  $47 + 12 = 59$  and  $59 - 12 = 47$
- Multiplication as repeated addition  
e.g.  $3 \times 4 = 4 + 4 + 4$
- Division as repeated subtraction  
e.g.  $24 \div 6 = 24 - 6 - 6 - 6 - 6$ .

### How students read and visualise a word problem will affect how they write it symbolically.

For example, if the problem is: 'I shared 2000 apples equally among 50 people; *how many apples will they each get?*' Or: 'I shared 2000 apples in boxes of 40 and gave a box each to some people with none left over. *How many people were there?*' Students need to visualise what is happening and,

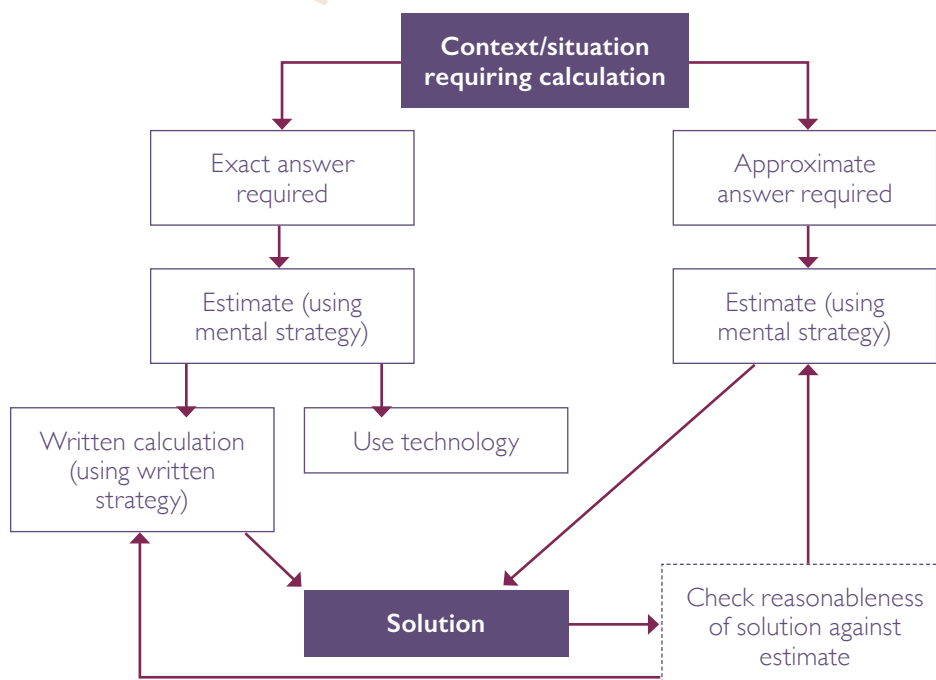
despite one question focusing on apples and the other on people, they will see that the apples are being shared among 50 people, either way.

A number of lessons can be delivered to teach students explicitly how to read and understand number sentences with a '+=' symbol in them. Students can be asked to both read and **visualise and/or draw** what they have understood from what they have read from the board and correct each other as needed. They can be asked to complete tables such as the following, working in pairs and groups, or developing similar tables for their peers to complete.

Problem in words	Written as	Read as
	$156 \div 3$	'One hundred and fifty six divided by three'
	$90 \div 15$	'Ninety divided by fifteen'

### Calculating

When confronted with a calculation that needs to be carried out, students should use their 'in-built calculator' (brain) *first*. Almost all single – and double-digit calculations can be done mentally using a mental strategy<sup>iv</sup>. If the numbers are too big to deal with mentally then students face the choice of a written strategy or a computational tool strategy. As the diagram that follows shows, these choices will depend on whether the answer required is exact or not.



Students need to be explicitly taught mental strategies. McIntosh (2004) suggests four generic strategies:

1. *My method* (students describe orally the strategy they used and how successful it was; this allows them to hear the different strategies used by others).
2. *How else?* (students devise a range of calculator strategies which result in the same answer; this allows students to do mental calculations at their own level of confidence).
3. *How will I calculate?* (students choose a suitable calculation method and give reasons for their choice; these can be mental, written and calculator).
4. *What's related?* (students recognise calculations that are related to the one they might be doing e.g.  $30 + 40 = 70$  is related to  $300 + 400 = 700$  and  $31 + 39 = 70$ ).

Strategies for teaching these and other mental computational skills can be found in McIntosh (2004).

As the magnitude of numbers students are working with increases, students will find that most calculations are best done using mental strategies to estimate, followed by some form of digital technology to compute. **The estimate is significant in providing students with a predicted magnitude for their solution.**

For example, in calculating  $351 + 28$  they might **estimate** first by thinking:

- *Three hundred and fifty add twenty eight is 378*

In **calculating** the exact answer they might think

- *Thirty five tens and two tens is thirty seven tens, one and eight is nine*

Thirty seven tens and nine ones is 379

If they need to support this thinking by writing things down as they go, they might write:

- $350 + 20 = 370$
- $1 + 8 = 9$
- $9 + 370 = 379$

This written support is entirely appropriate.

Similarly for multiplication, in **estimating**  $34 \times 207$ , they might think:

- *Thirty lots of 200 is 6 and three zeros*
- *6000; my answer will be a bit more than that because I've rounded both numbers down, ignoring the four and the seven*

In **calculating** the exact answer they might use a calculator, grid or long multiplication method and write:

	30	4
200	6000	800
7	210	28

$$\begin{array}{l}
 30 \text{ lots of } 200 = 6000 \\
 \text{Four lots of } 200 = 800 \\
 30 \text{ lots of seven} = 210 \\
 4 \text{ lots of seven} = 28
 \end{array}$$

**TOTAL 7038**

For **dividing** quantities they would still estimate based on their understanding of place value and operations.

For example, in dividing 562 by 18 they might **estimate** first, thinking:

- *Five hundred and sixty divided by about 20 is (five in every hundred which is 25) and three more is 28.*
- *If they are expecting a number of this size they would then likely use a **calculator** to obtain the correct answer and check its reasonableness by comparing their calculated answer with their estimated answer.*

Students should be encouraged to develop their own **informal strategies**; teachers need to model 'talking aloud their thinking' and 'jotting down numbers and words as they go' so that students see that 'this is how you do it'.

These informal strategies do not preclude standard algorithms. However, students should be taught that standard algorithms are only ONE way to do calculations. **Students should be encouraged to compare their informal methods with standard methods and see which they prefer.** Some students prefer to work with unmarked number lines for example, and teachers who predominantly use that method often find that many of their students do also.

Students can compare the setting out of various methods and talk about the advantages and disadvantages of each. The emphasis should be on understanding how and why the methods work. If they are using a method and *relying on the memorisation of each step in the method to make it work*, then this is unhelpful. Remember, an algorithm doesn't necessarily tell you 'where the student is going wrong'; errors may just be an indication of where their memory 'let them down'.

### **Solving problems and checking calculations**

Teachers need to really understand what a 'problem' is in the context of mathematics and/or numeracy. A problem is generally *a situation described in a written context requiring mathematical computation*. To solve such a problem, the strategy described in earlier sections, is ideal.

Students must first comprehend the problem and then restate it in their own words. The link between literacy and numeracy is at its strongest in problem solving since students need to *comprehend the context* by first understanding *the words*.

By far the **biggest challenge** that students face in solving mathematical problems is writing a number sentence to represent the calculation required. That is why the problem-solving model described earlier places so much emphasis on *clarifying the context*.

Clarifying is occurring when students visualise, and write or say in their own words – or paraphrase what they understand from situations and contexts. Visualising can help students see what is happening. Students need to practise writing number sentences for situations by working in small groups or pairs. By doing this they have the advantage of hearing all the different approaches that other students have; this can force cognitive conflict to challenge their thinking. Arguments can result which are healthy; students who have to defend their decision-making have to use appropriate language and really need to understand what is happening. Teachers do well to model this behaviour to students. They can play at being 'devil's advocate' to prolong the arguments and thus force more and more students to come to an opinion and to hear that *it is OK not to know the answer immediately*. By thinking through a problem and engaging in *struggle*, students learn and develop growth mindsets.

You might develop a table such as the following initially with students, using your own modelled thinking to show students how it's done, and then share the strategy before having students work in small groups to complete their table. When all groups have finished still more conflict can occur when groups share their entries and justify them in front of other students.

Problem	Paraphrased or visualised	Mathematics required in words	Symbolic representation
There were 25 673 people at a soccer match. One half of them supported the home side. About how many people supported the away team?	25 673 people at a soccer match. Roughly half are the away team, half are the home team  How many support the away team (estimate is enough)	Find about half of 25 673	$25\ 600 \div 2 =$

The **second biggest challenge** students have in dealing with problems is in sequencing the steps to be taken when there is more than one. This involves understanding the context, and thinking through what is happening in a logical way so that the steps become clear in their own mind; and having a strategy to write down the solution to the first part, then using that solution in solving the second part; then the third part, and so on. The same strategy used above will also work for teaching students how to solve problems with two or more stages.

Problem	Paraphrased and/or visualised	Mathematics required in words	Symbolic representation
Eight people share a prize of \$7450. They keep \$250 each and give the rest away to a charity. How much do they give to charity altogether?	Eight people have \$250 each and give the remainder up to \$7450 away	Work out what 8 lots of 250 is and <b>then</b> take the answer away from 7450	$8 \times 250 = \text{something}$ $7450 - \text{something} = ?$

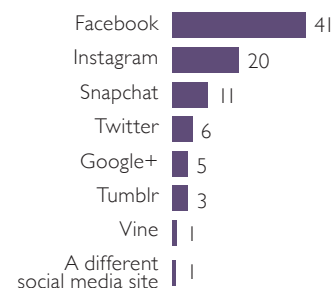
We must teach students these strategies if they are to successfully solve word problems. They need to *read* the words, *comprehend* their meaning and show this by visualising, drawing and paraphrasing. They then need to *draw out the maths* by breaking the questions into steps and writing a number sentence for each.

**Some skill problems and word problems of increasing difficulty (7–10) requiring calculations:**

- Frank and Eileen's house was built in 1856. How old was their house in 2014? Explain or show why.
- Which two odd numbers greater than 1, have a product of 173? Why?
- The richest man in the world, Carlos Slim, is purported to be worth 53.5 billion dollars. Bill Gates is second with a fortune worth about 53 billion. How much richer is Slim than Gates? (Write your answer in both standard form and scientific notation.)
- Which of these numbers is a multiple of both 3 and 13: 36? 313? 360? 3333? 3131? Why?
- Is  $25 \times 43$  equal to  $25 \times 4 + 25 \times 3$ ? Why or why not?
- Rod has six times as many blue pens as black pens and three times as many textas as black pens. How many textas does Rod have? Explain why.
- Is  $30^2$  the same value as  $15^2 \times 2^2$ ? Why are they not the same? Explain your answer.
- Which of the following expressions has the same value as  $30^3$ ?  $90^2 \div 3$ ;  $3 \times 3 \times 3$ ;  $9 \times 3^2$ ;  $3 \times 10 \times 10 \times 10$ . Give reasons for your thinking.
- Are these numbers in increasing order:  $\frac{3}{8}$ , 0.38, 38%,  $3^8$ ? Explain your answer using a number line.
- Noah shared 30 health bars with his friends at lunchtime. He gave James three bars less than he gave Keenan. He gave Dylan six more bars than James. He ended up having three times as many bars as Keenan. How many did Noah have in the end?
- In 2012 the population of Tanzania (in Africa) was 48 million while that of Spain (in Europe) was 46 million. The projected population in 2050 of these countries is current times 2.875 and current times 1.04 respectively. What are the 2050 projected populations and which country's population is growing at the fastest rate?
- Fingernails grow at the rate of about  $7.14 \times 10^{-3}$  metres per day. How many metres is that in a week? A year?
- One billion is a thousand million. What is 80 billion written in scientific notation?
- Consider the following graph and answer the questions:
  - What social media do most teenagers in the US currently use?
  - What does '41' represent?
  - What ages are the teens?
  - How big was the sample of teens used?

**Facebook, Instagram and Snapchat Used Most Often by American Teens**

% of all teens who use \_\_\_\_\_ most often



Note: \*Don't use any responses not shown.

Source: Pew Research Center's Teens Relationships Survey, Sept. 25-Oct 9, 2014 and Feb. 10 Mar. 16, 2015 (n=1,060 teens ages 13 to 17).



## Money

Sample activities requiring money calculations can be developed to address both the 'Understand and use numbers in context' strand and the 'Estimate and calculate' strand of the Numeracy continuum (Estimating and calculating with whole numbers element). These activities focus mainly on students having to choose which operator to use, depending on the context, to determine change from transactions (usually subtraction); and to perform these calculations using mental, written and digital technologies, estimating their answer first.

### Simple activities requiring money calculations

- Franci had \$15. With it she bought ten iTunes songs with each one costing \$1.20. How much change did she get?
- James made a phone call to a friend that lasted 29 minutes. The cost for the phone call per minute was \$0.79. About how much did the phone call cost? Write in words how you calculated this.
- Scott makes home-made soap. At the markets he made \$54 selling six bars of soap which were all priced the same amount. How much money will he make next week if he sells 15 bars of soap?
- Nancy and Ayesha entered a fun run and each collects some money. Nancy collected \$55. If Nancy had collected five dollars more she would have collected five times as much as Ayesha. How much did Ayesha collect?
- Sarah's petrol gauge in her car indicated that she had one quarter of a tank of fuel remaining. She then filled her tank up and it cost her \$45 at a cost of \$1.49 per litre. Roughly how much petrol will Sarah's fuel tank hold when it is full?
- Ross was given a \$20 voucher for his birthday. He bought six apps for his smart phone costing \$1.79 each. Write an expression indicating how much money he has left to spend (do not calculate the answer).

### Percentage, profit & loss

Possibly the most common financial activity needed by students as they become adults is that of determining the 'best buy' or 'best value for money'. This activity is shown as the key numeracy requirement in level 5 for Years 7 and 8 students. In Australia these decisions surround

anything ranging from a bottle of shampoo to a jacket to wear to the school formal.

In order to determine the better buy students need to use proportional reasoning. Proportional reasoning is about the relationship between the whole and its parts. This will be discussed in detail in Section C: Using fractions, decimals, percentages, ratios and rates. However, since financial profit and loss, best buys, and interest rates charged invariably require percentage calculation, it will also be briefly discussed here.

Percent means 'out of one hundred' and percentages are therefore another way of representing hundredths. Students should be explicitly taught the link by completing tables such as:

Number	Fraction	Hundredths	Percentage
0.03	$\frac{3}{100}$	3	3%

Percentages can mean different things in different contexts, and this is generally why students often find them difficult to understand. They are used:

- to represent fractions (as shown in the table)
- to show percentage increase or percentage decrease
- to compare numbers (in a part-whole fraction sense)
- as a statistic (e.g. the national unemployment rate)
- as a function (e.g. discounts and interest rates)

Students will generally be familiar with the use of percentage through the download bar on their computer which indicates the percentage of a file being downloaded. They should be able to describe how much of their file is downloaded, using statements such as: *one half/quarter of the file is downloaded; there's only 10% of the file left to download*, and they can see what that means visually:



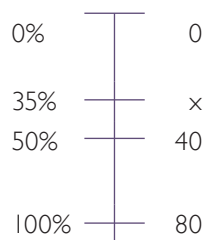
65%

If students understand the connection between percentage and fraction, they should be able to estimate results from percentage calculations using fractional equivalents as benchmarks: 25% for one quarter; 50% for one half; 75% for three quarters; and so on. They should use these measures as *benchmarks*.

Most students can find a percentage of an amount by representing it symbolically. For example:

Find 35% of 80.

We estimate first: *Think: 35 percent is more than one quarter of 80 but less than half of 80, so it's more than 20 but less than 40; about 30.* Visually:



Symbolically: '35%' can be written as  $\frac{35}{100}$ , 'of' meaning multiply, and '80' as 80 whole numbers:  $\frac{80}{1}$

$\frac{35}{100} \times \frac{80}{1}$ ; we might use a calculator:  $35 \times 80 \div 100 = 28$

Since 28 is close to our estimate, we are confident we are correct.

Note the steps we used<sup>v</sup>:

1. Write down the problem in words.
2. Estimate by visualising the **part** we are wanting to find and the **whole** amount.
3. Write the problem symbolically.
4. Calculate and compare our answer with the estimate.

**Profit made** is the amount gained on a transaction, **loss made** is the amount lost on a transaction. Students need to understand what **profit and loss** actually are and connections to real life possibilities are needed for them to learn to paraphrase profit and loss situations.

Have students give you examples if they can; some will no doubt have helped family members sell items on the web and will be familiar with both concepts in actual dollar amounts, if not as proportional amounts. For example:

- Frankie bought a bicycle pump for \$15 and sold it for \$25. How much profit did he make?
- Steve bought a bike helmet for \$60 and sold it for \$45. Did he make a profit or loss? How much was this?

Give students these types of questions initially to become familiar with the profit and loss concepts. Next discuss the percentage increase and percentage decrease; they need to find the actual dollar increase or dollar decrease, and express that as a percentage of the selling price i.e.

$$\frac{\text{Change in cost (i.e. increase or decrease amount)}}{\text{Original amount}}$$

For question 1 above:  $\frac{\$10}{\$15} = 10 \div 15 = 0.67$   
(2 decimal points);  $\frac{\$10}{\$15} = 67\%$

For question 2 above:  $\frac{\$15}{\$60} = 15 \div 60 = 0.25$ ;  
 $\frac{\$15}{\$60} = 25\%$

#### Some questions for percentage, profit & loss

- Mary-Jane wants to buy some new gym shoes in the latest colours. They are marked at \$210 but she is given a discount of 20%. How much is the discount worth and how much will she pay if she decides to buy them?
- Liela bought some new jeans at a sale which advertised '10% of everything storewide'. Write a mathematical expression for the amount she paid for the jeans she bought if they were originally marked at \$50.
- Simon sold an old pair of joggers on ebay for \$160. He originally paid \$120 for them. What percentage profit did he make?
- Milan advertised his bike on the Internet for \$1200. He paid \$850 for it three years ago. He sold his bike for \$600. Did he make a profit or a loss on the sale and what percentage was this?

### Percentage: best buys

'Best buys' usually involve calculations of money amounts. However, they also require some understanding of measurement attributes since the concept of *value for money* is a comparison of rates: cost per weight; cost per area; cost per length; cost per litre; and so on.

Start by comparing the same quantity of the attribute and comparing price only: *Which is the better buy: 200 grams of brand A costing \$4.50 or 200 grams of brand B costing \$3?* Students can clearly see that brand B is the better buy simply by doubling the quantity bought (assuming the quality of the product is the same). In most best buy calculations there is an assumption made about identical quality, but this should be discussed with students; can we always make this assumption and what else might we consider?

The example above is also a good one to start with because students can see that they are dealing with a product and a cost. Gradually increase the level of difficulty by changing the attribute amount through doubling or halving: *Which is the better buy: 100 grams of brand A costing \$4.50 or 200 grams of brand B costing \$3?* Students will calculate this by doubling the amount of brand A so that they can compare the same amount of both brands; or they can halve the amount of brand B so that they have a consistent unit, making comparison easier. Here, they can either compare 200 grams of each product or 100 grams of each product.

More challenging: by having students adjust their quantities so that the amount of each brand is the same; usually the *smallest common amount (the unit)*. *Which is the better buy: 300 grams of brand A costing \$4.50 or 200 grams of brand B costing \$3?* 100 grams of brand A is  $\frac{1}{3}$  of 300 so it will cost \$1.50. 100 grams of brand B is half of 200 so it will cost \$1.50 ... therefore they cost the same amount.

Still harder: Use the same method (i.e. adjusting the attribute amount until you can price the **smallest common amount** or **unit price**). *Which is the better buy: 375 grams of brand A costing \$4.50 or 150 grams of brand B costing \$3?*

25 grams of brand A is  $25 \times 4.5 \div 375 = 0.3$   
(30 cents)

25 grams of brand B is  $25 \times 3 \div 150 = 0.5$   
(50 cents)

### Some questions: better buys

Which is the better buy:

- 2L milk for \$3.80 or 1.5L milk for \$2.70?
- 500 g butter for \$3.80, 250 g butter for \$1.85, or 1 kg butter for \$5.90?
- A USB stick with 8 GB costing \$5.30 or a USB stick with 16 GB costing \$7.80?
- 1L juice for \$2.15 or 375L juice for \$1.10?
- 350 ml shampoo for \$4.50 or 500 ml for \$7.40.

Students should always be encouraged to do these calculations mentally since they will not need exact answers in most cases when considering the better buy. If you want them to use written methods you need to make the contexts more relevant e.g. that they only have a certain amount to spend.

### Budgeting, interest and financial plans

Students in Years 9 and 10 are fast nearing the age where they need to be **budgeting and evaluating financial plans** to support their specific, individual financial goals. Level 6 of the Numeracy continuum highlights this as the key numeracy activity for using money for these students. Many of them – particularly in Year 10 – will have employment during out-of-school hours and will have disposable incomes.

They need to make informed choices about their spending but also need to plan and budget ahead, particularly if they are paying taxes, either PAYG or annually. They will more than likely have bank accounts and have their weekly or fortnightly pay being deposited in these. They might, for the first time, come across **interest earned** on savings and the high rate of interest charged on credit card debt. Clearly they need to understand this concept, and even use their knowledge of interest on savings to help generate greater income for items they are saving for, such as for digital technologies.

### Simple interest

As students take on more responsibility for managing their own finances, there will be times when they might consider borrowing money to purchase something. They will need to understand **interest charged** by the lender or **paid** by the borrower for the privilege of borrowing money. You might introduce

simple interest by providing students with a list of contexts and have them work in pairs to determine whether each context relates to interest paid or interest earned.

In calculating simple interest, the following formula is used:

**Simple interest = Principal x rate of interest x time (in years)**

Since the rate of interest is expressed as a percentage, students need to learn to estimate

simple interest by mentally calculating:

*Rate for every hundred dollars of the amount, times number of years.*

For example, if they want to borrow \$300 and the interest rate is 4%, they would think:

*Four out of every hundred is \$12 for every year of the loan.*

They might practise this in pairs, given a table such as:

Client	Principal	Interest Rate p.a.	Time	Interest Paid or Earned
John Smith	\$600 borrowed	3%	2 years	
Alan James	\$820 banked	4.20%	3 years	
Sarah Smart	\$450 borrowed	6%	6 months	

Most interest situations are embedded in contexts that require students to read carefully and determine the principal, rate, and length of time from the words. For example:

*Jake wants to borrow \$150 for a new surfboard. If he borrows the money from his parents they won't charge any interest but he has to pay it off over 12 months. If he borrows it from a bank they will charge 12% interest per year, and if he borrows it*

*through a payday loan he has a year to pay it back at an interest rate of 11000%. Which is Jake's best option and why?*

Have students work in pairs or groups, clarifying the situation and determining how they will set out their work and which calculation model they need to use to make a case for the best option. Their work might look like:

<p>Mum &amp; Dad: <math>\\$150 \times 1 \times \frac{1}{12}</math></p> <p><math>\\$150 \div 12 = \\$12.5 = \mathbf{\\$12.50}</math></p> <p><b>He will pay \$12.50 per month for 12 months = \$150</b></p>	<p>Bank: <math>\\$150 \times 12\% \times 1</math></p> <p><math>\\$150 \times 0.12 \times 1 = \mathbf{\\$18}</math></p> <p><b>He will pay \$18 interest for the year plus the principal = \$168</b></p>	<p>Payday loan: <math>\\$150 \times 11000\% \times 1</math></p> <p><math>\\$150 \times 110.00 \times 1 = \mathbf{\\$16500}</math></p> <p><b>He will pay \$16500 interest for the year plus the principal = \$16650</b></p>
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**Other activities** might include having students research online the *simple interest rates for borrowing* of a range of banks and lending institutions, and also the *interest paid rates* by the same organisations. Are they the same? They should also look up fast facts on payday loans: [www.responsiblelending.org/payday-lending/tools...fast-facts.html](http://www.responsiblelending.org/payday-lending/tools...fast-facts.html)

### Compound interest

To teach students to calculate **compound interest**, teach them to add the interest earned each year to the principal (this becomes the principal for the next year) and calculate the

interest of the new principal e.g. compound interest for three years is:

Year 1: Interest calculated on initial principal.

Year 2: Interest calculated on new principal (initial principal plus interest earned in year 1).

Year 3: Interest calculated on new principal (year 2 principal plus interest earned in year 2).

### Simple financial plans

Students can be given the opportunity to work independently or in small groups to make simple plans to support their financial goals. They should learn to use spreadsheets to manage their

finances over time, and include interest earned and charged whenever relevant. Hypothetical situations might even include them determining interest earned by lending their finances to friends in order to maximise earning – this would of necessity include discussions about the risks of doing this, and could therefore be linked to Section E: Interpreting statistical information.

Students might record their monthly income in a spread sheet and use it to calculate the amount they will have by the end of the year. Teach them to use the  $\Sigma$  (sum) function to calculate the total.

Have students compare their monthly earnings with their yearly totals. Encourage them to think about changes such as:

- If, on top of your earnings, your parents gave you an additional \$50 for your birthday and Christmas, what difference will this make to your yearly total? How might you show these anomalies on your spread sheet?
- If your earnings were cut back by \$15 a month what difference would that make to your total?
- If you were told you had to pay \$20 per month towards your phone plan (or pet food), what difference would that make to your total?
- If your earnings dropped by half the amount every week, what total would you get for the year?
- If your best friend earned \$12 per week less than you, would you have more or less than her/him in three months? How much less? In eight months? How much less?

Ask questions about their pocket money like:

- If you gave \$5 every week to a charity, how would this affect your: a) monthly, and b) yearly, total?
- What other payments might you have to make regularly out of your earnings?
- What other additional amounts that are not regular, might affect your monthly totals?
- What other payments are *irregular* that you know you need to save for?



### Some activities for budgeting

- Gemma earns \$30 per hour at normal time and \$35 per hour for overtime. How much will she earn in total during a week when she worked five hours of normal time and three hours of overtime?
- Franka worked for eight hours at her normal pay rate. She also worked an additional five hours on the weekend at double her normal pay rate. She was paid a total of \$396. What is her normal pay rate per hour? Write an expression you would use to calculate her total pay for the 13 hours she worked.
- Aza has \$390 in the bank. He needs another \$30 to pay for a game console. The bank is paying him 3% interest per year on his savings. Will he earn enough in interest during the next year to make up the shortfall he needs? Why or why not?



### Monitoring and assessment

In order to monitor the learning against what students are expected to learn, teachers need to refer back to the intended learning in the *Australian Curriculum*. They need to continue to be very clear about what their precise goals are and share those with students.

To assess the deep understandings of mathematics, as described above, teachers must ensure that they have taught those understandings to all students using a high expectations approach and differentiation strategies as described in *Good Teaching: Differentiated Classroom Practice*. This may include the use of extending or enabling prompts for tasks/problems to broaden access to the learning.

A good source of assessment questions for students in Years 7–9 is some of the questions in past Years 7 and 9 NAPLAN Numeracy test papers. The questions generally align closely with the definition of numeracy and:

- provide no hints about the mathematics students should choose to use
- are for the most part written in contexts so that students have to understand these before choosing the mathematics to use

These questions can be accessed from the NAPLAN Toolkit site <http://naplan.education.tas.gov.au> (staff only) and through the IMPROVE formative assessment site <http://www.improve.edu.au/> (This free site enables teachers to create and use online assessment tests and quizzes for students with quality assured questions).

## Links across the curriculum

Other curriculum areas provide contexts for the application of mathematics. Some of these contexts **demand** understanding of the mathematics concepts in order to understand their content. For example, students won't understand timelines in History if they don't understand how number lines work and ordering of numbers into the thousands. Teachers can either show students how useful their maths learning can be in these contexts or use the contexts as **opportunities** for *teaching* the mathematics concepts.

In the increasing specialisation of secondary teaching areas, including option or elective areas from Year 9 such as Technologies, the importance of numeracy demands and opportunities takes on a richer meaning for teachers. Who better to demonstrate the applications of mathematics in real-life contexts than the teachers of these learning areas? Teachers in such areas need to draw the links by asking students: *Will some maths help here?* Or, *what maths are we using here?*

### Examples from other learning areas

Students often apply their mathematics learning in other subject areas without knowing it. Options teachers don't have to be maths teachers but must be prepared to teach their own particular form of using the maths in these contexts. These can be context-specific numeracies; those that have developed for a particular purpose and context, and with a particular application.

In **Foods and Hospitality**, students are often required to increase quantities for catering and bulk cooking. They do this by multiplying amounts. For example, if a recipe is designed for four people, how can it be *scaled up* for 20 people? 200 people? This is an opportunity for teachers to discuss multiplication and its application to a real-life situation. Similarly, recipes can be *scaled down* – usually requiring division – for smaller groups of diners.

In **Languages**, students learn to count and use different pattern systems for place-value understandings. Comparing these with those of our own cultural heritage can help consolidate mathematical understandings learned in mathematics class. Students use numbers in other languages when talking about age, height, telephone numbers, addresses, and so on.

In **Science**, students are learning about phenomena on Earth including seasons and eclipses and that they are caused by the positions of the Sun, Earth and Moon. Large numbers are needed for comparing and ordering, and scientific notation is essential for these representations. A sense of the size of these distances will support deep understandings about the magnitude of the solar system and beyond. Students are engaged in collecting data through measuring which often requires heightened levels of accuracy to a number of decimal places, often with some rounding. They have to make judgements about 'how accurate' that are appropriate to the task and should be engaged in these discussions with their teachers.

You would point out to them that 'we are using some maths here'.

In **History**, students need to be able to sequence historical events, developments and periods. Periods may overarch events and hence students need to see history in years and in blocks of time. Connections can be made with a number line to show individual years but also with different periods of years, making parallels with sets of numbers including integers or 'negative numbers between 100 and 650' and showing these connections on a number line indicating '100 BC – 650 AD'; this will also aid their understanding of integers.





## Questions for reflection

How do I understand the benefits for estimation and mental calculation and being able to partition (See Years K–2 and 3–6 booklets for more detail on partitioning) numbers? How might I help students understand this?

How do I model the importance of using a mental strategy as my first choice in calculation?

How do I explicitly teach students mental strategies? Do I model their use by sharing my 'thinking aloud' to explicitly model problem solving?

How much time do I focus on mental strategies in my class? Is it enough when I consider that this strategy is the one that my students will need the most in life?

In what ways do I discuss and ask whether we need an accurate answer or whether an approximation will suffice for the context?

Do I understand why teaching standard algorithms might be harmful for my students? How does this influence my planning and teaching?

Why do I teach standard algorithms? Is this reason valid? Is it because it's what I know best?

Do I understand the mathematics concepts that underpin the content in this section? How are the topics connected? How might I learn more about this?

Do I spend time teaching standard algorithms in the belief that my students will understand mathematics more if they can do them?

How might I design assessment tasks to focus less on the use of standard algorithms?

How do I model and teach the valuable skill of estimating with money?

## B. Recognising and using patterns and relationships



### Key messages

Some teachers may wonder why *recognising and using patterns and relationships* is important in numeracy or for that matter, mathematics. Mathematics is the science of patterns<sup>vi</sup> and patterns are used to help us organise and make sense of the world in which we live. We therefore need to be able to **generalise** about these patterns rather than having to study each separate pattern. 'Mathematics brings to the study of patterns an efficient and powerful notation for representing generality and variability, and for reducing complexity – algebra'.<sup>vii</sup>

There are **patterns** in our number system (0, **1**, **2**, **3**, 4, 5, 6, 7, 8, 9, 10, **11**, **12** ...) for example, that help students learn to count. There are patterns in our lives (Monday, Tuesday, Wednesday ...) that help us organise our lives and our work. We can learn the words of songs more easily for example, by considering the patterns in the words. In mathematics, we can learn to count more easily by recognising the patterns that exist in our number system. Patterns can be **related** to each other in ways that can help us study the patterns regardless of the specific elements in the pattern. This is why **being able to generalise** is so important; it is about asking: 'What is happening here that has nothing to do with the *specific* numbers used?' In a pattern like 4, 8, 12, 16 ... we can say: *It is growing by four each time* and this generalisation would be same if the number pattern were 3, 7, 11, 15 ... or 29, 33, 37, 41 ...

In the K–2 resource booklet, Section A, we learned that  $3 \times 4 = 4 \times 3$  and that this relationship always works no matter what numbers are used. This can therefore be **generalised** as:  $a \times b = b \times a$  where 'a' and 'b' can represent any number. This is an example where students can engage in algebraic thinking.

This type of thinking leads to an understanding of *functions* which are the basis for formulas used to determine phone plans, taxi fares, rental charges, energy consumption and other costs in our lives. Being able to understand these is an important part of numerate behaviour since numeracy involves recognising and understanding the role of mathematics in the world.

Whereas patterns and relationships in Years K–2 focus mainly on objects, drawings, sounds and other representations, patterns and relationships in Years 3–6 focus on **patterns represented with numbers**, and in particular, on identifying and representing trends within them. In Years 7–10, students use algebraic rules to represent patterns and relationships, and use the relationships to identify trends. They can then use these trends in a practical way to facilitate their planning.

vi Steen, 1988

vii A National Statement on Mathematics for Australian Schools, 1990.

## Links to the curriculum

Patterns occur in all learning areas. In the mathematics curriculum the content descriptors in the Number and Algebra strand describe what students are expected to be able to do by the end of Years 7–10.

Relevant Australian Curriculum: Mathematics Content Descriptors			
Year 7	Year 8	Year 9	Year 10
<p><b>Patterns &amp; algebra</b></p> <p>Introduce the concept of variables as a way of representing numbers using letters</p> <p>Create algebraic expressions and evaluate them by substituting a given value for each variable</p> <p>Extend and apply the laws and properties of arithmetic to algebraic terms and expressions</p>	<p>Extend and apply the distributive law to the expansion of algebraic expressions</p> <p>Factorise algebraic expressions by identifying numerical factors</p> <p>Simplify algebraic expressions involving the four operations</p>	<p>Extend and apply the index laws to variables, using positive integral indices and the zero index</p> <p>Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate</p>	<p>Factorise algebraic expressions by taking out a common algebraic factor</p> <p>Simplify algebraic products and quotients using index laws</p> <p>Apply the four operations to simple algebraic fractions and numerical denominators</p> <p>Expand binomial products and factorise monic quadratic expressions using a variety of strategies</p> <p>Substitute values into formulae to determine an unknown</p>
<p><b>Linear and non-linear relationships</b></p> <p>Given co-ordinates, plot points on the Cartesian plane and find co-ordinates for a given point</p> <p>Solve simple linear equations</p>	<p>Plot linear relationships on the Cartesian plane with and without the use of digital technologies</p> <p>Solve linear equations using algebraic and graphical techniques</p>	<p>Find the distance between two points located on a Cartesian plane, using a range of strategies, including graphing software</p> <p>Sketch linear graphs using the coordinates of two points</p> <p>Find the midpoint and gradient of a line segment on the Cartesian plane using a range of strategies including graphing software</p>	<p>Solve problems involving linear equations, including those derived from formulas</p> <p>Solve linear inequalities and graph their solutions on a number line</p> <p>Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology</p> <p>Solve problems involving parallel and perpendicular lines</p>

Relevant Australian Curriculum: Mathematics Content Descriptors

Year 7	Year 8	Year 9	Year 10
Investigate, interpret and analyse graphs from authentic data	Verify solutions by substitution	Sketch simple non-linear relations with and without the use of digital technologies	Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circle and exponentials using digital technology as appropriate  Solve linear equations involving simple algebraic fractions  Solve simple quadratic equations using a range of strategies

Extracts from Australian Curriculum: Mathematics Achievement Standards

Students represent numbers using variables  They connect the laws and properties for numbers to algebra  They interpret simple linear representations and model authentic information. Students solve simple linear equations and evaluate algebraic expressions after numerical substitution  They assign ordered pairs to given points on the Cartesian plane	They make connections between expanding and factorising algebraic expressions  They simplify a variety of algebraic expressions  They solve linear equations and graph linear relationships on the Cartesian plane	They expand binomial expressions  They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment  They sketch linear and non-linear relations	They solve problems involving linear equations and inequalities  They make the connections between algebraic and graphical representations of relations. Students expand binomial expressions and factorise monic quadratic expressions  They find unknown values after substitution into formulas  They perform the four operations with simple algebraic fractions  Students solve simple quadratic equations and pairs of simultaneous equations
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Australian Curriculum Numeracy Learning Continuum

Level 5	Level 6
<b>Recognise and use patterns and relationships</b> Identify trends using number rules and relationships	Explain how the practical applications of patterns can be used to identify trends

Identifying trends using 'rules' that describe relationships requires a basic understanding of how quantities relate to each other. Similarly, rules that describe relationships (e.g. those devised by a telco company to determine mobile phone plans) are generally derived by others; a potential user would need to know how to substitute a number into a given rule to generate a solution. Students need to know how a rule might be determined, how it works and how it can be used to generate different solutions depending on numbers substituted into them or 'inputs'. This does not require deep algebraic understandings, but rather a capacity to think algebraically and use some algebraic skills.

## Planning

Teachers and educators use the information in the 'Links to the curriculum' table in order to plan. They need to understand precisely what the intended learning means. Focusing on the verbs is important and this will be described in more detail below. The verbs capture what students need to *know, do and understand* with respect to this element by the end of the particular year of the *Australian Curriculum: Mathematics*, or the level in the Numeracy continuum. For example, students should be able to *Verify solutions using substitution* by the end of Year 8 if they are to achieve what is expected in Mathematics; and *Identify trends using number rules and relationships* by the end of Year 8 if they are to be numerate.

Planning also requires teachers to have some understanding of what their students already know, understand and can do, so that they can build on this learning with the intended 7–10 learning. The 3–6 resource booklets indicates what they might be expected to know, do and understand if they have had access to teaching that attends to the expected standards of the *Australian Curriculum* in these years. However, it is not a good idea to assume that all 7–10 students have had this access. It will be useful to obtain a copy of the 3–6 resource booklets both to remind yourself of these earlier expectations, and to be familiar with the teaching knowledge and strategies suggested for younger students, in order to differentiate teaching for secondary students if necessary. The challenge for teachers of students in the middle years is to re-teach basic concepts in ways that are appropriate for adolescent learners; you wouldn't use the same methods for Years 7–10 students that you would have used for primary children.

The activities in the following 'Putting it into practice' section describe the sorts of learning opportunities teachers need to create to develop the stated learning, both for numeracy and the mathematics that underpins it. The activities are not intended to be a comprehensive list, but describe the type of activities that are required – your students might need more or less reinforcement depending on where they 'are at' in their current learning. Note that the activities focus on the deep learning required for *numeracy* outcomes rather than all the mathematics outcomes in the table.

In the *Recognising and using patterns and relationships* strand of the Numeracy learning continuum, the main understandings Year 7–10 students need to learn and develop are:

### In **Patterns and algebra**

1. What variables are.
2. How to form and simplify algebraic expressions.
3. How to write and use algebraic expressions to find solutions of equations (including proportional relationships).
4. How to substitute values into rules/formulae to determine unknown quantities and verify solutions.

### In **Linear and non-linear relationships**

5. How to identify variation, represent it visually, and interpret visual representations.



## Putting it into practice

### I. Teaching students about variables

Many secondary students develop misconceptions in algebra as a direct result of the ways in which the concept of a variable are taught.<sup>viii</sup>

There are two key understandings in formal algebra that are critical for all students to understand what algebra is and what it is used for:

#### a. Variable quantities

Students should have been introduced to the ideas of generalising in their studies of numbers and operations and their properties in the K-2 and 3-6 resource booklets. For example, in learning about partitioning, addition and subtraction and their relationship to each other, they analysed examples such as:

$$3 + 4 = 7; 7 - 3 = 4$$

$$8 + 2 = 10; 10 - 2 = 8$$

$$16 + 5 = 21; 21 - 5 = 16$$

... and were consequently able to say that: *something general is happening here that has nothing to do with the operations I'm using or the numbers I'm using.* They might even express this as: *If I add two numbers together and get a total, then that total subtract one of the numbers will give me the other one.* If they accept this (and they should be encouraged to try and find numbers for which this doesn't work), they are able to write this generalisation using symbols representing variable quantities:

$$\text{If } a + b = c \text{ then } c - b = a \text{ and also } c - a = b.$$

Similarly, having worked with patterns, they might be able to generalise a pattern and say, for example that: *each term is the sum of the two previous terms.*

Students should have many opportunities to express these generalisations using words so that they can see the advantages of representing them using symbols. For example, if they say: *If I add two numbers together and get a total, then*

*that total subtract one of the numbers will give me the other one,* they can see themselves the benefits of being able to use a *shorthand* to write:

If  $a + b = c$  then  $c - b = a$  and also  $c - a = b$  knowing that  $a$ ,  $b$ , and  $c$  represent any combination of three numbers that have that relationship and that they can be whole numbers, integers, fractions and decimals.

#### b. Concatenation

Concatenation is an important part of the semantics of algebra. It is the accepted convention of placing a number quantity alongside a variable quantity and using this to represent a product. For example,  $2h$  stands for ' $2 \times h$ ' and represents 'two lots of whatever number  $h$  represents'. The practice of omitting the multiplication symbol probably originated from the perceived confusion that can develop if 'x' is confused with the variable quantity  $x$ . Variables are generally italicised in print for the same reason.

When introducing variable quantities teachers need to give students opportunities to verbalise this understanding through their reading of terms. This will strengthen understanding and assist in avoiding misconceptions that arise through students believing that they are merely working with letters in maths.

### 2. How to form algebraic expressions and equations from words, and simplify them

Your next challenge is to ensure students learn to consolidate the number of similar terms; this task becomes easier if students deeply understand variable quantities and concatenation. For example, students can see that  $3m + 4m$  is 7 lots of whatever number 'm' represents rather than  $mmm + mmmm$ .

In order for students to simplify expressions such as  $3z^2$  they need to deeply understand the rule of order of operations and the concepts of indices. These have been discussed in the 3-6 resource booklet so will be assumed here, but teachers will need to ensure this has been learned before attempting to simplify algebraic expressions. By thinking: *'I can't add  $3a$  to  $5a^2$  because  $a$  represents one number and  $a^2$  represents a different number:  $a \times a$ '*, students show they understand the concept of variable amounts.



### Activities to consolidate these skills and understandings

- Write expressions on the board for students to evaluate, using their calculator if necessary. Assign values for the variables such as  $c = 12$ ;  $d = -3$ 
  - iii.  $3c^2 + 4d - 8d$
  - iv.  $9d^3 - 53c + 61c - 3d^3$
  - v.  $c \times (d - 268c) + 4d$
  - vi.  $-5c \div 14d + 12c - -11d$
  - vii.  $(45 \div 5) \times 12c$ .....and so on
- Expressions such as those above can be developed to represent real life situations; students learn how to do this through practice and through assigning variable quantities to unknown amounts. For example:

*Luke earns \$36 for washing 12 cars. How much will he earn for washing z cars?*

- Students first need to read the problem carefully to first determine the unknown quantity they are trying to find. In writing an expression representing this situation, students should think about the unknown quantity: *what is it?* Here, they are trying to determine the total \$ amount for washing z cars. Teach them to assign a variable amount:

*Let 'n' be the number of cars.*

Next, teach students to verbalise what they are trying to find before attempting to write it symbolically.

Total amount earned = amount earned for washing one car x total number of cars

Symbolically,  $n = (36 \div 12) \times z$

So, Luke earns a total of  $3z$  dollars.



### Some activities to practise forming and simplifying expressions representing real situations

- When solving equations such as those above, it is always useful to substitute the solution (the calculated value of the variable) back into the original equation to check whether you are right or not. This substitution skill is also useful when determining or checking costs or amounts owed, particularly when businesses use formulae in their calculations and provide these to you.
  - Julie has \$49 which is more than twice the amount James has. How much does James have? Justify your answer.
  - Christa spent \$35 at the grocery store. This was seven dollars less than three times what she spent at the bookshop; how much did she spend there?
  - Sam spent \$100 on books. This was  $h$  dollars less than five times what he spent on his lunch. How much did he spend on his lunch?
  - The sum of two consecutive numbers is 99. What are they?
  - A class of 50 students is made up of two groups; one group has eight less students than the other. How many students are in each group?

### 3. How to write and use algebraic expressions to find solutions of equations (including proportional relationships)

Solving given equations requires students to consolidate and simplify expressions – as described above – and to then obtain a numerical value for the variable quantity. Students in Years 3–6 are solving equations when they find the value obtained through an operation. For example, they solve for  $x$  when they compute  $x + 3 = 8$ . They might think: *What number plus three is eight?* The number must be 5. They have actually used the part-part-whole relationship to solve this equation since  $8 - 3$  is their solution. They have *isolated the variable*,  $x$ , by subtracting 3 from both sides of the equation.

This is where their understanding of how expressions are formed and the relationship described by the '=' symbol are so important. Students who believe the '=' symbol represents an operation (i.e. 'Do something now to get an answer' as opposed to representing an even or equivalent 'balance' between the left and right hand sides of the equation) come unstuck. The importance of teaching the '=' symbol as one of *equivalence* as opposed to an *operator* has been explained in the 3–6 resource booklet. When working with algebraic terms students need to be comfortable with the notion of equivalence; if they aren't they are likely to *close* terms rather than work with them in order to satisfy their need to *get an answer*. This incorrect belief in the '=' symbol as an operator is demonstrated by students who believe, for example, that  $2a + 3b = 5ab$ . (See Good Teaching 3–6 for more information on misconceptions about = as an operator)



#### Some activities to consolidate this understanding

- An expression equivalent to  $3f \div 2 + 15f + 12 =$
- An expression equivalent to  $45 + 3z - 29z + 34 =$
- An expression equivalent to  $5g^2 - 7g + 24g - 12 - 6g^2 =$

**Solving by isolating the variable** is an essential skill and is taught by teaching students to 'unwrap the variable' one operation at a time in the order in which it was 'wrapped'<sup>ix</sup>. For example, if  $3x + 5 = 7$ , then to isolate  $x$  you need to reverse the order of the operations used on it to place it where it is in the current equation: these were *multiply by 3 and then add 5*. To inverse these, in reverse order: subtract 5 and divide by 3. You then need to perform these operations on the other side of the equation: *7 subtract 5 and divide the result by 3*. So  $x = \frac{2}{3}$ .



#### Some activities to practise solving equations

- $4c - 7 = 13$ ; what is  $c$ ?
- $0.6a + 17 = \frac{3}{4}$ ; what is  $a$ ?
- $58.3 = 19k \div 12$ ; what is  $k$ ?
- Audrey takes part in a lapathon to raise money. She has two sponsors; her father donates \$3.20 for each lap she completes and her boyfriend gives her \$60 in total. How many laps does Audrey need to run to raise \$200? (You might need to consider whether a full lap needs to be completed in order to obtain each \$3.20).

In particular, this method is useful for isolating the variable when proportional quantities are involved (and will be used extensively in Section C for this reason).

For example, to isolate the variable in the following expression:

$$\frac{x}{5} = \frac{8}{13}$$

*unwrap*  $x$  by multiplying by 5 and doing this to the other side of the equation:

$x = 8 \times 5 \div 13 = 3.077$ ; *Estimate the answer and explain your thinking*: We expect  $x$  to be just a bit more than one half of 5 just as 8 is just a bit more than one half of 13, so 3.077 is what we expect. Students should always be taught to estimate first!

This technique is useful and students will be able to generalise that given any two equivalent proportional amounts expressed in this way, isolating the variable always requires you to multiply the two given amounts that are diagonal (here, 5 and 8), and divide by the remaining one (here, 13).

Similarly, if  $\frac{7}{x} = \frac{12}{17}$  we estimate the value of  $x$  first and expect it to be more than 7 but less than 14 since 17 is more than 12 but less than double 12, or 24. Here,  $x = 7 \times 17 \div 12$  which is 9.92. This procedure is called **cross-multiplying**. Be sure you teach students why it works using unwrapping; don't just teach it as a method without the understanding required as this could create misconceptions!

### Activities to practise isolating the variable and solving the equation

- $\frac{3}{16} = \frac{d}{25}$ ; what is  $d$ ?
- $\frac{9}{f} = \frac{249}{78.6}$ ; what is  $f$ ?
- In Australia, distances on roads are measured in kilometres whereas in the USA, road distances are measured in miles. If 8 kilometres is about the same length as 5 miles, about how many miles is 240 kilometres?
- Financially, one Euro is worth about 1.5 Australian dollars (at time of writing). About how many Euros would AUD\$500 be worth?

### More challenging ... substituting one equation into another

- Sometimes in forming equations, students will need to work with more than one equation. They may have an equation with three unknown values in it and need to work with the context to simplify and get only one variable quantity. For example, consider:
  - George is  $\frac{4}{5}$  the height of Sam and Fred is  $\frac{1}{2}$  the height of Sam. Fred is 20cm shorter than George. How tall is Sam?

It helps to first draw (or at least visualise) this relationship. There are three people involved and you want to find the height of Sam (let his height be 't') who apparently is the tallest.

Fred is  $\frac{1}{2}$  t, George is  $\frac{4}{5}$  t

Fred is 20cm shorter than George: ( $\frac{4}{5}$  t - 20)

So, ( $\frac{4}{5}$  t - 20) =  $\frac{1}{2}$  t

Therefore  $\frac{4}{5}$  t -  $\frac{1}{2}$  t = 20

$$\frac{3}{10}t = 20$$

$$t = 66.67\text{cm}$$



$$S = t \quad G = \frac{4}{5}t \quad F = \frac{1}{2}t$$

Students should check: does this seem right? According to our diagram the three boys are in the proportions given.

### 4. How to substitute values into rules/formulae to determine unknown quantities and verify solutions

As noted earlier, when solving equations such as those above, it is always useful to substitute the solution (the calculated value of the variable) back into the original equation to check whether you are right or not.

As was also indicated, it is essential that students understand *concatenation* when substituting values into a formula or equation. Most formulae omit multiplication symbols and students need to know this. Practise with real-life formulae that are relevant to students.

#### For example:

- The cost of a taxi-fare is determined using the formula: Cost of taxi = \$3.20 + \$1.40 per km.
- Calculate the cost of catching a taxi home after a party where the distance is about 28 km.
- Discuss the formula with students: 'What might the \$3.20 be for and why is it the same regardless of how far you want to travel?'

**Teach students to be methodical in their substitutions and to use words if they help, not just numbers. Model the use of brackets and explain to students why/how they help and that they also work to indicate multiplication instead of the 'x' symbol.**

Substituting known values:

$$\begin{aligned} \text{taxi cost} &= \$3.20 + \$1.40 (28) \\ &= \$3.20 + 39.20 \\ &= \$42.40 \end{aligned}$$

Make sure you teach students to estimate solutions since they are unlikely to want to or be able to perform a written calculation if they are at a party! Talk about rounding \$3.20 to \$3, \$1.40 to \$1 and 28 to 30. Discuss the fact that you have rounded two numbers down and one number up; are you likely to have an over-estimate or an under-estimate?



### Some activities to practise substituting values into formulae

- When making a pot of tea, a general 'rule of thumb' is used to indicate how many teaspoons of tea are needed in the pot: 1 tspn + 1 tspn/person. How many teaspoons would you need for eight people?
- The cost of renting a car with a particular hire-car company is given by the rule: \$35 + \$5/km. What might the \$35 be for? Will the company round the number of kilometres up to the nearest kilometre or down? Why? Calculate how much it could cost to hire a car and travel 289 km.
- Roasting time for a leg of lamb is given by the rule: 40 minutes per kg plus 30 minutes. What assumptions do you think are made in developing this formula? What might the 40 minutes be for? How long would you need to cook a 2.25 kg leg of lamb?
- A petrol can that Greg keeps in his car has a diameter of 30 cm and a height of 40 cm. If volume of a cylinder is found using the rule  $V = \pi r^2 h$ , how much spare petrol could Greg carry in the can?
- The slope of a straight line is given by the formula: slope (gradient) = rise/run. What will the slope of a line be, if it has a rise of 4 metres and a run of 7.35 metres?
- Find the daily and yearly cost of using your electric kettle if the utility rate is 11 cents per kW/h and the rule for calculating the daily consumption is  $(W \times T) \div 1000$ , where wattage (power rating) (W) of your kettle (written on it) is 1500, and T represents the time in hours that you use your kettle every day, which is about one hour. How much would it cost to run your fridge over a 24-hour period if its power rating is 440W?

### In linear and non-linear relationships

#### 5. How to identify variation, represent it visually and interpret visual representations

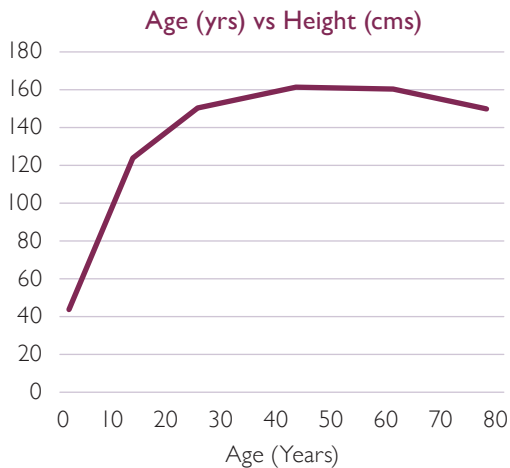
Variation in a relationship describes the extent to which the relationship deviates from its normal state or its expected trend. For example, if two quantities are directly proportional (i.e. increase or decrease at a constant rate) their relationship is linear (a straight line). If there is variation in this relationship (i.e. something happens so that the relationship is no longer constant) their relationship is no longer linear; it varies.

For example, our height varies over time; our height may increase at a constant rate for some periods of our life, but generally we have spurts of growth as we develop between childhood and adolescence, and finally our height reaches a peak, remains constant for a long period of time, and may even decrease as we get older. In this relationship, time increases at a constant rate with no variation, but our height varies over time. Since both of these quantities are positive we can represent this relationship in the positive quadrant of a Cartesian plane. Data points for example, might be:

Age (yrs)	1	10	20	30	40	50	60	70	80
Height (cms)	45	124	150	155	160	160	160	155	150

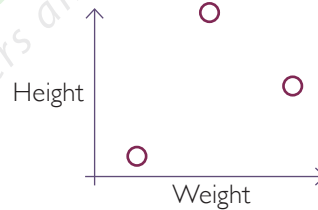
The exact data points aren't really important when we are describing the variation in the relationship. We are generally more interested in how the data is **trending**; is it increasing, decreasing or remaining steady or constant. This was described in the 3–6 resource booklet in Section 2. The relationship might be shown visually for example, as that shown here representing the data above.

We might **describe the variation** by saying 'the height increases steeply while we are young, remains the same for many years, and then decreases in old age'. This variation can also be identified in the data points in a table, but is often more clearly seen in the visual representation.



**Focus questions** include:

- Which quantity should go on the horizontal axis? Why?
  - Can we see a relationship? Is it linear? Will it remain linear? Why or why not?
  - Does it make sense to join the data points and make a line?
  - Can we use the line to predict for amounts we don't have data for? How confident can we be about extrapolating the data? Interpolating the data?
- Have students consider any 'habits' they might have that involve amounts of time taken each day. For example, amount of TV they watch, amount of time spent on digital devices, amount of time spent playing sport. Have them draw a table representing their data and plot a graph to represent their data. They should examine both the data points and the visual displays and write statements about their data; any patterns they notice – is the time spent increasing, decreasing, steady? Are there 'weekend bumps'? Do students notice any trends in their data? They should be encouraged to share their data with peers and ask others to interpret their data to see if others notice things that they don't.
  - Give students graphs showing qualitative relationships such as:



The graph shows three different people with different heights and weights. Students can be given clues such as the following to determine who is where on the graph:

- a. Fred is lighter than Sam.
- b. Fred is taller than John.
- c. Sam weighs more than John.

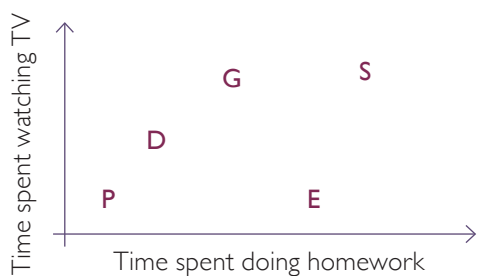
Students can work in pairs to make up their own and give them to other students to solve.



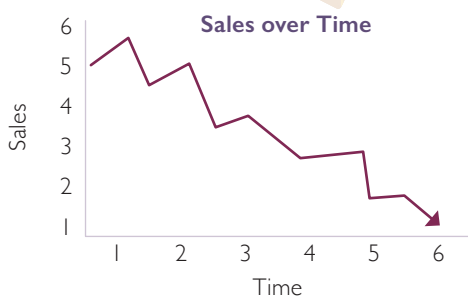
**Some activities to support the development of these understandings<sup>x</sup>**

- Have students generate data point tables and graphs through investigating relationships between:
  - number of cars and number of tyres
  - number of people and number of fingers
  - number of people and number of shoes
  - number of hamburgers and cost
  - number of cows and number of legs, and so on.
- Give students a similar graph with data points marked and ask questions about the variation e.g. time spent watching TV and doing homework last weekend:
  - Who watched the most TV?
  - Who did the least homework?
  - Who spent more time doing homework than Pam?
  - Who spent less time watching TV than Gilbert?
  - Who spent more time watching TV than Pam and less time doing homework than Gilbert?

- Sam spent more time watching TV than Gilbert but less time doing homework than Ewan. Shade a rectangle that indicates where Sam could be.
- Write three more relationship statements from this graph.



- In the following graphical displays, which ones show relationships that are increasing, decreasing or steady?



Have students engage in considering the patterns and trends in their own lives both currently and into the future, with a view to making predictions that might support them in their planning. They might want to start collecting data to support this activity, including such variables as:

- Mobile phone use per month
- Amount of data they download monthly
- Amount they spend on entertainment per month/week
- Amount of TV they watch
- Amount spent on bus transport, and so on



## Monitoring and assessment

Most of the ideas presented here in activities can be used to assess the understanding of pattern and relationships. Note that understanding of these concepts needs to include higher-order reasoning about pattern and relationship; assessment must go well beyond the skills needed for algebra. The activities suggested and provided above provide reliable assessment tasks. Students should be able to make conjectures about patterns (their own and others found in the media, for example) and predict what will happen to a pattern as it trends upwards or downwards, based on the cycle in their pattern and the variables they might know or surmise contribute to it.

Monitoring these would involve keeping and reviewing data points over time, looking for 'bumps' in variation, and reasoning about why these might be occurring. They might consider whether their trend becomes 'smoother' over time to eventually eliminate the effect of the 'bump', or whether it has a lasting impact.

As well as using their own data, students should be able to substitute values into rules and formulae to plot their own visual representations of relationships. The syntax and semantics of formal algebra will help them to do this. Teachers should remember the desired numeracy goals for their students when teaching these; don't get caught up in teaching algebraic language for



their own sake. Many students become alienated from school mathematics in the middle years as a result of the way algebra is taught. They can perceive it as 'being taught how to move letters around their page' with no connection to their personal lives. Teachers can avoid this through teaching in practical and relevant contexts, such as those suggested above.

Many of the activities in this section can be adapted and used to assess students' understandings. Another good source of assessment questions for students in Years 7–9 is some of the questions in past Years 7 and 9 NAPLAN Numeracy test papers. The questions generally align closely with the definition of numeracy used in this resource booklet and:

- provide no hints about the mathematics students should choose to use
- are for the most part written in contexts so that students have to understand these before choosing the mathematics to use.

## Links across the curriculum

Other curriculum areas provide contexts for the application of mathematics. Teachers need to draw the links by asking students: *Will some maths help here? Or, what maths are we using here?*

Some of these contexts **demand** understanding of the mathematics concepts in order to understand them. For example, students won't be able to compare events in their lives with those of students in other cultures if they can't understand and describe the patterns in their own. For instance, their daily sequence of getting up, having breakfast, and travelling to school might not be the same for students from different cultures. Similarly they won't understand how different texts are put together unless they can recognise the common patterns in them. You can show students how useful their maths learning can be in these contexts and use the contexts as opportunities for *teaching* the mathematics concepts.

## Options/electives

Students often apply their mathematics learning in other subject areas without knowing it. Options teachers don't have to be maths teachers but must be prepared to teach their own particular form of using the maths in these contexts. These can be context-specific numeracies; those that have developed for a particular purpose and particular application.

In **Foods and Hospitality**, students may use 'rules of thumb' – some of which have already been explained above – including rules for making drinks e.g. *3 scoops of coffee for ½ a plunger pot*; or *bake meat for 30 minutes on high and then one hour on low for every kilogram of meat*. Teachers should describe these rules as: 'we are using algebra here'. Similarly many recipes are written that require oven temperatures in Fahrenheit rather than Celsius. Students need to know how to convert these but don't need to actually calculate, since a 'rule of thumb' may be sufficient.

In **Visual Arts**, students also use patterning and generalisation of patterns when they design beading sequences, painting and stitching patterns: *Will the shell pattern fit across the pillowcase opening? How might I be sure?* This would include measuring and dividing the entire length into shell sections to 'fit'.

In **Science**, when students are learning inquiry skills, they make predictions as a result of analysing patterns and relationships in a range of representations including graphs. They also analyse patterns and trends in their data, including describing relationships between variables and identifying inconsistencies and variation. Consider Ohm's Law (about the relationship between the voltage of a current and resistance) as a specific example for students studying high-level Science.

In **History**, students are learning about changing settlement patterns over time and resulting from natural and social phenomena; and migration patterns – do these result from common activities? Are there patterns in events leading up to migration 'waves', and so on?

In **English**, students are learning about patterns in different text types and the ways in which documents are put together following various patterns and formulae. Text types create expectations in readers who analyse these patterns. Titles and headings predict content of

books and other texts and the patterns within chapters can be used to predict what might be occurring at the beginning, middle or end of a chapter.

There are some more numeracy links across the curriculum described on the *Australian Curriculum* website.



## Questions for reflection

How would I help a colleague understand what number patterns are and why they are included in the Mathematics curriculum and Numeracy continuum?

Can my students describe a pattern? How might I support students who are finding this difficult and extend those who already know?

Do I expect my students to be able to show me one pattern in another form?

How might I support them to do this? What might I need to model?

How do I support my students to know the difference between a growing pattern and a repeating pattern?

How do I help my students to know and use the language they need to describe and compare patterns?

What explicit teaching is required to enable students to predict what will happen when they continuously add or subtract the same number? What about if they multiply each successive term by the same amount?

What assessment tasks would enable me to recognise when students can generalise about a pattern? What should I listen for?

How do I currently help my students understand trends? Could I use trends and predictions in my own life and model this to students?

## C. Using fractions, decimals, percentages, ratios and rates



### Key messages

The order of the words in this heading is significant. Decimals, percentages, ratios and rates are all different forms of fractions. They are also developmentally cumulative, so that students learn about *common* fractions first, then about *decimal* fractions, then about percentages – which are a particular type of fraction, having a denominator of 100; then they learn about ratios and rates. If teachers try to teach students about ratios or percentages before they have grasped the *concept* of fractions, they will not understand and will resort to learning ratios by heart to get answers right. Moreover, if students don't understand the connections *between* these concepts and learn them as unrelated ideas of content, they will lack the crucial deep understandings needed to work with these fractional representations.

The second thing worth noting in the heading is the word 'using'. **You can't use fractions if you don't first understand them.** So in order for students to choose to use fractions in situations that require them, they need to have a deep understanding of what they are, and what they can be used for. Many Australian students – including in secondary settings – don't really understand fractions. In fact, research<sup>xi</sup> and subsequent teaching revealed students in Year 8 who thought that a fraction was merely a different way of writing two whole numbers; separated by a 'funny little line'!

It is critical that the idea of **equal** quantities or parts is explicitly taught to students during early fraction lessons. This enables children and young people to understand that  $\frac{1}{2}$  for example, is 'one out of two equal amounts' rather than just two parts of the same whole. This understanding is critical for the understanding of proportional

reasoning and comparing fractional amounts such as  $\frac{3}{8}$  of one whole with  $\frac{4}{7}$  of another.

### Links to the curriculum

In Years 7–10 the focus of a fraction and its various representations is on *proportional reasoning*. The mathematical concept of proportional reasoning has been described as one of the mathematics concepts most often applied in the real world<sup>xii</sup>. It is therefore a major aspect of numeracy.

As mentioned in the Years 3–6 resource booklet, proportional reasoning is the ability to understand the relationship of a whole quantity to its parts, and situations of comparison. If not well understood, it can severely restrict the ability of students to successfully reason in a proportional way.

Proportional reasoning is complex; it requires students to use a broad range of mathematical knowledges and skills, including the ability to identify relationships between quantities and represent them symbolically. Because proportion is often taught as an isolated topic in the curriculum<sup>xiii</sup>, many teachers fail to appreciate the underpinning and essential conceptual understandings, and connections between these understandings, it requires.

Proportional reasoning:

- brings together all the mathematics ways of working
- is embedded in a range of different mathematics strands and topics due to the fact that rates and ratios require understandings in arithmetic, measurement, statistics, and chance
- requires facility with a variety of different number representations, including ratios, rates, decimal numbers, percentages and common fractions.

In examining the entries in the table below, teachers can see the fractional understandings required for the proportional reasoning outcomes of the Numeracy continuum.

xi Perso, T. (1989)

xii Lanius & Williams (2003)

xiii Dole (2010)

**Relevant Australian Curriculum: Mathematics Content Descriptors**

Year 7	Year 8	Year 9	Year 10
<p><b>Real numbers</b></p> <p>Compare fractions using equivalence. Locate and represent fractions and mixed numerals on a number line</p> <p>Solve problems involving addition and subtraction of fractions, including those with unrelated denominators</p> <p>Multiply and divide fractions and decimals using efficient written strategies and digital technologies</p> <p>Express one quantity as a fraction of another, with and without the use of digital technologies</p> <p>Round decimals to a specified number of decimal places</p> <p>Connect fractions, decimals and percentages and carry out simple conversions</p> <p>Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies</p> <p>Recognise and solve problems involving simple ratios</p>	<p>Investigate terminating and recurring decimals</p> <p>Investigate the concept of irrational numbers including <math>\pi</math></p> <p>Solve problems involving percentages, including percentage increases and decreases, with and without digital technologies</p> <p>Solve a range of problems involving rates and ratios, with and without digital technologies</p>	<p>Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems</p> <p>Apply index laws to numerical expressions with integer indices</p> <p>Express numbers in scientific notation</p>	

**Extracts from Australian Curriculum: Mathematics Achievement Standards**

Students express one quantity as a fraction or percentage of another	Students solve everyday problems involving rates, ratios and percentages	Students interpret ratios and scale factors in similar figures	
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**Australian Curriculum Numeracy Learning Continuum**

Level 5	Level 6
<p><b>Interpret proportional reasoning</b></p> <p>Visualise and describe the proportions of percentages</p>	<p>Illustrate and order relationships for fractions, decimals, percentages, ratios and rates</p>
<p><b>Apply proportional reasoning</b></p> <p>Solve problems using simple percentages, ratios and rates</p>	<p>Solve problems using fractions, decimals, percentages, ratios and rates</p>

Proportional reasoning is usually applied in problem solving. This capability in life is demanded in financial contexts, including profit and loss, determining 'best buys', determining percentage of a quantity and expressing one quantity as a percentage of another. Since money is included in Section A, these skills were discussed in that section.

Problem solving with proportional quantities is also required when comparing proportional amounts in quantities such as population sizes – in particular where numbers are very large or very small, and where comparison is better understood by comparing *proportions* rather than real numerical values. Expressing quantities using proportions also enables direct comparison (since proportions can be compared with other proportions) to provide greater meaning. Sometimes students fail to recognise proportion when they meet it in a context. It is important for teachers to support students to engage in *recognising proportion* as the first step to understanding the concept. For example:

#### Activity to understand proportional reasoning

- In the context of *area* in the measurement strand:



Which shape has more purple?

In an **absolute** sense the first shape does, however in a **relative** sense the second shape does. Have your students discuss this in groups; they can do this at any age.

In Years 7–10, the key numeracy ideas of fraction and hence proportion are:

- Interpreting and applying proportional reasoning understandings.
- Understanding ratios and rates and solving problems.
- Understanding the relationships between different fractional representations – including between common and decimal fractions, percentages, ratios and rates – enabling them to be compared and ordered on the same scale.

#### Proportional reasoning

A **proportion** is a part or quantity of a whole considered in comparison to the whole.

**Proportional reasoning** refers to the thought processes undertaken to arrive at statements about proportions, such as 'half as much as' or 'double the amount'. These statements provide more comparative detail than simply 'bigger than' or 'less than'. This indicates more than understanding comparison; it includes understanding the **context or situation** that the comparison is occurring in. That is why proportional reasoning is so important for numeracy.

Consider the following: *If one cat grows from 1 kg to 3 kg while another grows from 4 kg to 6 kg, which cat grew more?* In an absolute or additive sense they both grew the same amount i.e. 2 kg. However, in a relative sense the first cat clearly grew to a greater extent when we consider the weight gain in proportion to the original weight. Proportional reasoning has occurred to enable the thinking that leads to this conclusion; the thinking involves visualising the amount of growth (in size) with respect to the amount of original size, without necessarily focusing on the actual numbers.

Proportional reasoning is not something you either can or cannot do. Rather, it is a *capability developed over time*, through reasoning.

Proportional reasoning is often about **comparing a proportion to its whole amount as well as comparing it to a proportion of another whole**. This requires mental gymnastics! It is complex and therefore requires deep understandings about fractions and the wholes they relate to. In calculating, we mostly try and reduce the wholes to the smallest **common unit**.

Other examples of when proportional reasoning is needed, include:

- Determining the 'best buy' when 1 kg cost \$3.50 and 1.5 kg costs \$5.20.
- Sharing two oranges between three people.
- Working out whether '4c per litre off' is better than 5% off the total when purchasing petrol.
- Determining which car is faster: one that travels 30 km/hour over 70 km or one that travels 45 km/hour over 90 km.

One of the biggest difficulties for students (and adults) is recognising *what* is being compared with *what*. To reason about proportions, students need to:

- Understand the different types of numbers (and representations of numbers) and how they relate to one another.
- Recognise multiplicative relationships in a range of different contexts and situations.
- Be comfortable using non-whole numbers (e.g. common fractions and decimal fractions) as operators. For example,  $3.25 \times 7$  and  $2/5 \times 29$ .
- Recognise and understand the different types of language used for operations in real-world situations. For example, that  $1/2 \times 16$  is *1/2 of* 16, and that  $2.5 \times 30$  is two and one half *times or lots of* 30. Students should also be able to estimate the solutions for these calculations and know what they are expecting.

## Planning

Teachers use the information in the 'Links to the curriculum' table in order to plan. In particular they need to understand precisely what the intended learning means. Focusing on the verbs is important and this will be described in more detail below. The verbs capture what students need to *know, do and understand* with respect to this element by the end of a particular year of the *Australian Curriculum: Mathematics* or the relevant level in the Numeracy continuum. For example, students should be able to *Solve problems involving percentages, including percentage increases and decreases* by the end of Year 8 if they are to achieve what is expected in Mathematics; and *Visualise and describe the proportions of percentages* by the end of Year 8 if they are to be numerate.

The activities in the following 'Putting it into practice' section describe the sorts of learning opportunities teachers need to create in order to develop the stated learning, both for numeracy and the mathematics that underpins it. The activities are not intended to be a comprehensive list but describe the type of activities that are required – your students might need more or less reinforcement depending on where they 'are at' in their current learning.



## Putting it into practice

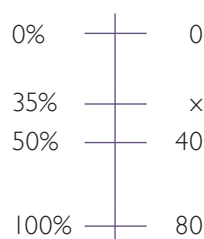
### Percentage as proportion

Since percentages were needed for the section on money (see Section A of this resource booklet), understandings and strategies for teaching them have been previously described. In that section, students were taught to find percentages of quantities and how to express a quantity as a percentage of another.

A visual method was outlined to demonstrate a way of students being able to see and compare the quantities they are working with e.g.

Find 35% of 80 *Think: thirty five out of one hundred is what out of eighty?*

Students are using two scales: the percentage scale (out of 100) and the number scale with 80 being the complete (whole) number: 80 is 100%. Visually, this can be shown as:



Encourage students to place any known 'benchmarks' on their scale; here, they know that 50% is half of 80 which is 40. They can also include 25% which is 20 if they wish.

Estimating first: *Think: 35 percent is more than one quarter of 80 but less than half of 80, so it's more than 20 but less than 40 .....  $\approx 30$ .*

Students solve their problem by representing it using fractions:  $\frac{35}{100} = \frac{x}{80}$

Cross-multiplying (multiply the two numbers diagonally opposite each other and divide by the remaining number):  $35 \times 80 \div 100 = 28$ .

Comparing with their estimate allows students to conclude they are likely to be correct.



### Some sample question types:

Have students work in pairs to solve questions such as those following (make sure they use a visual model and estimate first):

- What is 25% of 90?
- What is 36% of 70?
- 58% of 390?
- 22% of 84?

### Common fractions as proportion

In Years 1–6, (see resource booklets K–2 and 3–6) students learned that one half is 'one out of every two equal parts' and that this understanding could be represented visually, in words and in symbols. They learned to **benchmark** fractions between those they can 'see', including whole numbers, to help them to estimate the relative sizes of fractions.

They also 'counted with fractions' using number lines initially and then visualising fractions to help them picture  $\frac{4}{5}$  as being between  $\frac{1}{2}$  and 1 and  $\frac{3}{5}$  as being slightly larger than  $\frac{1}{2}$ . Similarly, knowing that  $\frac{3}{3}$  and  $\frac{4}{4}$  and  $\frac{5}{5}$  and  $\frac{10}{10}$  are all different names for 1, they learned the fundamental understanding of **equivalent fractions**; that fractions can represent the same *amounts* even if they have different names. They could picture  $\frac{1}{4}$  on a number line for example, and know that it is half of  $\frac{2}{4}$  which is  $\frac{1}{2}$  of  $\frac{4}{4}$ . By deeply understanding fractions in this way they were able to **estimate fractional quantities** before applying written methods when calculating with them.

### Equivalent fractions – benchmarking to assist estimating and calculating proportional equivalents

Consider the following examples:

- Students can read  $\frac{3}{4} = \frac{?}{5}$  as: *Three quarters is what out of five?* Doing this will help them to understand what they are trying to find. They should be able to reason that: *half of 5 is 2.5 so ? must be about half way between 2.5 and 5, say 3.8.* This provides an estimate they can compare their answer with.
- Being able to estimate in this way is rigorous and requires proportional reasoning. If students are unable to do this as they are developing their understanding then their

calculation is meaningless; it becomes a mere procedure. Spend time encouraging students to estimate – model it by talking aloud your thinking and have them share their reasoning in a similar way with each other. They should be required to write their estimate down before they attempt to calculate formally i.e.  $\frac{3}{4} = \frac{?}{5}$ ;  $\approx 3.8$

Using the cross-multiplying method shown in Section A, students can then calculate:

$$3 \times 5 \div 4 = 15 \div 4 = 3.75$$

They compare this with their estimate and can be confident they are correct.



### Some activities to practise this skill

Have students work in pairs to solve the following, estimating first:

$$\frac{4}{7} = \frac{?}{8}$$

$$\frac{?}{4} = \frac{13}{15}$$

$$\frac{7}{1} = \frac{11}{18}$$

$$\frac{2}{3} = \frac{9}{x}$$

$$\frac{35}{45} = \frac{k}{30}$$

$$\frac{y}{12} = \frac{3}{11}$$

### Ordering fractions: different representations of fraction

Students need to be taught the different representations of fraction. Fractions can be represented as common fractions, decimal fractions, percentages, quotients, ratios or rates. Many of these can be represented as numbers and can be shown as positions on a number line. Students need to convert them to common fractions before doing this and should be shown how.

Fraction representation	Example	Part	Whole
Common fraction	$\frac{1}{2}$	1	2
Decimal fraction	0.5	5	10
Percentage	5%	5	100
Quotient	$1 \div 2$	1	2
Ratio	1:2	1	3
Rate	2km/hr	2km (for every)	1 hour

- Some sample question types:
  - Place the following in increasing order: 65%, 6.5,  $\frac{6}{5}$ , 0.065,  $5 \div 6$
  - Place the following in decreasing order: 23%,  $\frac{2}{3}$ , 2.3, 0.203,  $3 \div 2$
  - Place the following numbers on a number line: 0.43, 43%,  $\frac{4}{3}$ ,  $\frac{3}{4}$ , 0.4,  $3 \div 4$

Putting numbers on a number line where all gradations are not indicated requires measurements, partitioning and spatial reasoning as well as proportional reasoning. For example:

On the number line shown, where would 11 go?



Students need to visualise gradations. This requires mentally breaking up the number line into equal portions between 6 and 10 and then extending the units beyond 10 by one equal portion to 11. Determining the equal portions requires proportional reasoning.

Similarly, consider the problem: *If five cans of drink cost \$5.75, how much do six cans cost?*

These type of activities require students to understand equal portions, to be able to partition into equal units, and to use equivalence strategies. Again, they are working with two different quantities – drinks and money. Students should draw the situation to help them to visualise these quantities:



They might even put these quantities in a growing pattern table (see Section B Year 3–6 resource booklet) to see the growing and dependent relationship between them:

Number of cans	1	2	3	4	5	6
Cost	\$1.15	\$2.30	\$3.45	\$4.60	\$5.75	x

Determining the equal portions (cans) and equating each to a unit price, and then extending these beyond the base number (5) by adding the unit cost *once* for the sixth can, requires proportional reasoning. Students can connect number, algebra and fraction/proportion in these activities.

Activities such as these are essential to support students in moving from additive thinking to proportional thinking. As students progress from year to year teachers should increase the variety of contexts students deal with, and the complexity of the situations.

### Contexts and involving proportion

As stated earlier, one of the greatest challenges facing students confronted with many of these problem types, is that they fail to recognise that the situation is one involving proportion. Teachers should support them to see the multiplicative relationship between quantities, as shown in the table above; they need to see that the cost *depends* on the number of units. In a situation where different unit rates exist for the price of objects – such as six pens cost \$3.60 and

12 pens cost \$4.80 – students can see that the relationship between the pens and the cost is not proportional between the two quantities of pens, even though it might be proportional between one lot of pens and their cost.

Some types of questions to practise identifying and writing quantities as proportion include:

- If two litres of petrol costs \$3, how much will 35 litres cost?
- If I drive 95 km in one hour, how far will I drive in three hours, given that I am driving at the same rate?
- If six kilograms of sausages cost \$13.40, how much will I pay for 2.5 kilograms?
- If Sam walks 3 km/hr, how far will he walk in 7 hours at the same rate?

### Rates, ratios and proportions

A **ratio** is a comparison between two or more quantities. It can compare:

- a. a part of a quantity to a part of another quantity e.g. 3 boys to every 5 girls
- b. a part of a quantity to all of another quantity e.g. 4 boys to every 12 students
- c. two quantities of different measures e.g. km/hour; these type of ratios are called rates. (Note that 'km' and 'hour' are *measures*. The ratio of boys to girls is not a rate since 'boys' and 'girls' are not measures but categories).

For (a) above, students should learn that *the unit amount is the total of each category*: in the example given, the unit is 8 students, 3 of whom are boys and 5 of whom are girls. So for every 8 students 3 are boys and 5 are girls. Encourage students to identify this unit before they attempt to determine an unknown quantity from the ratio. Explicitly teach them that the order of writing the quantities is important; for 3:5 'to' is represented by ':' and read as 'for every'.

Students should identify the quantities using headings and write the numbers they are given:

<u>Boys</u>	to	<u>Girls</u>
3	:	5

three boys for every 5 girls (total unit = 8 students)

If there are 24 students in the class, there are three groups of 8. For every 8, 3 are boys and 5 are girls so there must be 9 boys and 15 girls.

9 : 15 total of 3 × 8 students is 24 students.

Some problems to identify and solve ratio quantities:

- If the ratio of boys to girls in a class is 5 to 7, how many boys will there be if there are 48 students?
- If the ratio of boys to girls in a school is 5 to 7, how many boys will there be if there are 56 girls?
- In a litre of made-up cordial, 100 ml is cordial and the remainder is water. How much cordial is there in a made-up bottle of cordial containing 3.5 litres?
- A medicine requires the user to dissolve 5 ml in 200 ml of water for every 20 kg of the patient's weight. How much medicine is needed for a person weighing 55 kg? How much liquid will they need to drink to take this dose?

A **rate** compares two quantities of different *measures*. For example, kilograms per hour, litres per kilometre, dollars per kilogram. In teaching students to write rates they are generally taught to use the '/' symbol and to read it as 'per'. To foster their understanding of a rate, in the same way that we earlier taught younger students to read and understand a vinculum (fraction line) as 'for every', we teach students to read the '/' as 'for every' also.

For example:

- students read and understand  $\frac{3}{4}$ , as 'three out of every four equal amounts'
- students read  $x \text{ dollars/litre}$ , as ' $x \text{ dollars for every litre}$ '

This will help them understand the symbol and also the rate.

A **proportion** is an equation stating two ratios as equivalent e.g. '\$1.30 per litre equals \$2.60 per two litres'. These proportions can be written as equivalent fractions:  $\frac{\$1.30}{1\text{L}} = \frac{\$2.60}{2\text{L}}$

If the quantities are changing at the same rate, such as the one above, they can be expressed in a table. Helping students to visualise such a table will support their capability to deal with simple proportion mentally.

Litres	1	2	3	4	5
Cost	\$1.30	\$2.60	\$3.90	\$4.20	etc

Note that the **unit rate** is important and many students will see that by organising their information in this way, that they can find the cost for any number of litres just by knowing the cost of one litre.

If they want to find out how much a certain quantity will cost or how many units cost a certain amount, they might also express this visually in the same way that percentage has been taught. For example: *if petrol costs \$1.30 per litre, how much petrol will I buy for \$72.80?*

Petrol (L)	Cost \$
X	\$72.80
1	\$1.30

Or, they might simply write:

Petrol (L)	Cost (\$)
X	72.8
1	1.3

The main thing is to determine what the quantities are and keep them separate.

Since both petrol and \$\$ are increasing proportionally, they can also write the quantities as fractions. As long as they keep all of the same quantities together (e.g. all the petrol quantity as the denominators of the fractions or all of the petrol quantities as the numerators of the fractions or all on the left or all on the right), it doesn't matter how they write the equation:

e.g.

$$\frac{x}{1} = \frac{72.8}{1.3} \text{ or}$$

$$\frac{x}{72.8} = \frac{1}{1.3} \text{ or}$$

$$\frac{72.8}{x} = \frac{1.3}{1}$$

Estimate first: 1 costs \$1.30; roughly how many \$1.30s are there in \$72? There will be less than 70 (if it were \$1/L) but more than 35 (if it were \$2/L), so about 50-ish.

They then solve using cross-multiplication: multiply the two values that are diagonally opposite and divide the answer by the remaining number:  $1 \times 72.8 \div 1.3 = 56$ .

Check with the estimate: *OK, it's what I was expecting.*

If students have difficulties working with equations and variables, keep it simple: use a question mark '?' for the unknown amount, have them keep the quantities separate under headings and teach the method of cross-multiplying: multiply the two values that are diagonally opposite and divide the answer by the remaining number:  $1 \times 72.8 \div 1.3 = 56$ , which they can do in one procedure on their calculator.

Learning the method without necessarily fully understanding why it works, may be useful for students who can understand the proportional thinking but can't cope with solving equations. To promote numeracy we want to promote their ability to demonstrate understanding of proportion.

Teachers should note the **importance of using the same unit** for the same measure or attribute when representing equivalent proportions. We can't compare apples with oranges, neither can we compare the number of kilograms with the number of grams, or

the number of metres with the number of kilometres, even though they are used to measure the same attribute. Before students can compare two proportions they need to ensure the base unit for each of the measures, are the same. For example, consider:

- One man drives at 50 km/hour and another drives at 27 m/sec. Who drives faster?

Here we have four different measures; two for distance, and two for time. We can't compare until we have only two: we need to change the distance measures to the same unit; either kilometres or metres, and the time measures to the same unit; either hours or seconds.

Changing distance measures to metres      Changing time measures to seconds

50 km = ? m

1 hour = ? secs

? = 50 000 m

? = 3 600 seconds

Proportions:

Distance (m)

Time (secs)

50 000

3 600

Comparing proportions: 13.88 m/sec compared with 27 m/sec

Clearly the second driver drives faster.

### Activities to identify and solve proportion problems

Have students work in pairs or small groups to discuss the proportionality – or not – of the following questions (you should encourage them to discuss assumptions or provisos they might add to make non-proportional relationships, proportional):

- Six chocolates cost 96c. How much do 13 cost?
- Rod and Frank have 30 CDs between them. Alex has three times as many CDs as Frank. How many CDs does each boy have?
- There are six oranges and 15 bananas in a fruit bowl. What percentage of these fruit are oranges?
- James has a large box of cereal with a mass of 375 grams priced at \$5. A smaller box has a mass of 150 grams and costs \$3.40. Which box is the best value for money?

There are **four main types of proportional relationships:**

1. **Direct proportion:** when two quantities change in the same direction (both increase or both decrease); most situations described above are examples of this type.
2. **Inverse proportion:** when one quantity increases in one direction and the other decreases in the opposite direction e.g. it takes three painters two days to paint a conference room; how long should it take one painter? To deal with these situations students work in the same way as with simple proportion examples but then write the reciprocal of the second proportional quantity.

For example:

<u>painters</u>	<u>days</u>
3	2
1	?

Since we expect the '?' to be more than 2, we write

3	?
1	2

(we have inverted the days)

Solving, we use the same cross-multiplying technique:  $3 \times 2 \div 1 = 6$  days; (this makes sense when we consider the situation).

3. **Complex proportion:** when proportions involve more than two variables, one of which is related in a positively proportional way and the other is related in an inverse way. For example, the concept of *density* in Science involves the relationship between the volume of an object (i.e. the amount of space an object takes up) and its mass (how heavy it is). The density depends on two variables: mass and volume. One object such as a fishing sinker for example, may take up the same amount of space as a piece of wood that is the same size but the sinker has a greater mass than the wood. Both the mass and the volume must be considered simultaneously to consider and discuss their densities; to understand why the sinker sinks and the piece of wood floats.

4. **In-part proportion:** when a relationship between two quantities involves a constant amount, *and* a proportional amount. For example, a function to calculate a taxi fare is an in-part proportion function since the total fare results from a constant amount (the flag fall which is the same regardless of the cost per kilometre or the distance travelled) and a proportional part which is determined by distance travelled times cost per kilometre (these functions were considered in Section B of this booklet).

#### Solving problems using rates, ratios and percentages

Have students work in pairs to solve the following examples of the types of problems they need to be able to do. Make sure they identify quantities, represent them in appropriate ways, and estimate their solutions first.

- There are 25 oranges and 35 bananas in a fruit bowl. What percentage of the fruit are oranges?
- James has a large box of cereal with a mass of 375 grams priced at \$5. A smaller box has a mass of 150 grams and costs \$3.40. Which box is the best value for money?
- Kiesha lives 2.45 km from school and she walks to school at a rate of 3 km/hour. How many minutes does it take Kiesha to get to school?
- A drawing of a dog (from above) is 12 cm long and 5 cm wide. If the dog is actually 50 cm long, how wide is the dog?
- Jack's model car is 8 cm long. His Dad's car is 3.5 m long. How many times as long as Jack's model car is his Dad's car?
- The fastest speed of a fancy sports car is 205 kilometres/hour. How fast can it go in kilometres/second? Metres/second?
- Scott filled his car up with petrol costing \$1.42 per litre and it cost him \$56. The petrol gauge in his car showed he had quarter of a tank when he arrived at the pump. How many litres does Scott's petrol tank hold when it is full?

- A pond holds 36 kilolitres of water. It takes 24 hours to fill. How much water will be in the pond after 15 hours? What is this amount in cubic metres?

Have students work in pairs to first consider different variable relationships (e.g. cost per page) and then write proportional questions relating to these rates for each other to solve. Have them determine the solution/s so that they can be involved in the estimating and marking of each other's work.

Other activities involve comparing proportions of cultural background represented by groups that make up the Australian population. Data can be obtained from the Australian Bureau of Statistics website. Teachers can ask students to make up similar proportion questions using information on the website, and/or design questions such as those that follow:

Study the table and answer the following questions:

Year	Proportion of Estimated Resident Population (ERP) of Australia born overseas	Number of people
2003	23.6%	4.7 million
2012	27.3%	6.2 million
2013	27.7%	6.4 million

- If, in 2012, 27.3% of Australia's ERP is 6.2 million people, what was Australia's total ERP population?
- What was the rate of growth in Australia's population between 2003 and 2012? Write a sentence to compare this with the rate of growth between 2012 and 2013.



- The proportion of the Australian population who were born in the UK decreased from 5.7% in 2003 to 5.3% in 2013. What was the rate of decrease per year?

Consider the information in this table:

Country of birth	Proportion of Australian population 2003 (%)	Proportion of Australian population 2013 (%)
New Zealand	2.1	2.6
China	1.0	1.8
India	0.6	1.6

- Which country had the greatest increase in proportion of population living in Australia during the ten years from 2003 to 2013?



## Monitoring and assessment

All the above tasks and problems can be used for assessment as well as teaching. Students can be asked individually to find solutions and to draw and write about how they know they are correct. You might also give them a question with a hypothetical student's response, and have them assess it: *Are they right? How do you know? How would you help them if they have made an error?*

A good source of assessment questions for students in Years 7–9 is some of the questions in past Years 7 and 9 NAPLAN Numeracy test papers. The questions generally align closely with the definition of numeracy used in this resource booklet and:

- provide no hints about the mathematics students should choose to use
- are for the most part written in contexts so that students have to understand these before choosing the mathematics to use

## Links across the curriculum

Some of the contexts across the curriculum **demand** understanding of the mathematics concepts in order to understand them. For example, students won't understand a simple recipe if they don't know what 'half a cup' or 'three cups' means. You can either show students how useful their maths learning can be in these contexts or use the contexts as **opportunities** for *teaching* the mathematics concepts.

**Foods and Hospitality**, provides a wide range of opportunities for teachers to support students explicitly in applying their mathematics. These might include:

- Scaling down recipes for a smaller group can often require *halving* or *thirding*. If quantities are given as fractions (e.g. half a cup) students are required to find *half of half*. They can discuss whether doing it 'by eye' is accurate enough – depending on the purpose and group they are cooking for – or whether they should calculate exactly i.e. how much error can I tolerate?
- In costing a dish, students would calculate prices of individual ingredients within a recipe to arrive at the cost (e.g. of a tart or cake) and then use fractions to calculate the cost of an individual serve.
- When determining equal shares of something they have made, students use proportional reasoning. For example, if three students make a Swiss roll 10 cm long and share it equally, how much will they each get? They might do this 'by eye' and know that they needed to cut it first into thirds and then cut their own third into slices to share with more people.
- Calculating the percentage of wastage and then converting to an actual cost requires reasoning about cost and percentage. For example, if bananas cost \$4 per kilogram but there is 30% wastage due to the weight of the skins, the actual cost of the banana flesh is what per kilogram?

In **Visual Arts**, students use proportions when mixing paint, dividing pages into quarters, and when calculating plaster to water ratios. These are all valuable opportunities to apply their mathematics learning and students should be both alerted to these and supported in gaining deeper understandings and skill levels with the numeracy involved.

In **Design and Technology (Wood and Metal)**, students use fractional applications when they:

- Use plans (for example, a plan for making a shelving unit may be drawn with a ratio of 5:1 or 1:10).
- Use fractions of millimetres when gauging screw sizes, or designing low tolerance fittings such as a dowel joint.
- Adjust a belt speed on a drill, using a speed-setting table (done to suit the bit being used or the material being drilled).
- Use a 'golden rectangle' or other proportions considered design-perfect.

In **Music**, students use fractional understandings when learning the values of notes and rests. As with symbolic fractional representations, the top number represents the 'count' of time (the number of a certain note value per bar) and the lower number represents the *type* of notes in the bar. For example, a time signature of  $\frac{3}{4}$  tells the reader that there are three of the type of note value per bar; the lower number (4) tells the reader that the three notes are crotchets (since there are four crotchets in a semibreve). A metronome is used to *measure* time in music. Note values are related to one another in proportions: one semibreve = 2 minims = 4 crotchets = 8 quavers = 16 semiquavers. So a minim for example, is played in half the time of a semibreve, and 16 semiquavers are played in the same time as a semibreve.

There are some numeracy links across the curriculum described on the *Australian Curriculum* website. Many **Science** topics in the curriculum such as density, molarity (a chemical measure of concentration), speed and acceleration, and force require knowledge and understanding about ratio and proportion. For example, the concept of density is applied when estimating the number of penguins in a huddle needed to reduce body temperature of each penguin; understanding why a dog or baby locked in a car on a hot day suffers

more than an adult in similar circumstances, requires proportional reasoning. Importantly, a lack of developed proportional reasoning can have life-threatening implications, for example, if incorrectly calculating the necessary dose of medicine or inaccurately mixing chemicals or pesticides.

In **Science**, students will need to know how to compare numbers including fractions, and put them in order. They might make up solutions that require proportions such as '15g sugar for every 2L of water'. They learn that different types of chemical reactions are used to produce a range of products and the reactions can occur at different rates.

In **History and Social Sciences**, students might be comparing composite populations of countries such as 'one third of the population are Asian' or 'two fifths of the country is populated', and so on. They consider changing profiles in population proportions resulting from changing migration patterns or settlement patterns over time and from one place to another. Students work with scale when drawing and reading maps.

In **Technology**, students might be involved in cooking or making craft which requires fractional measurements such as 'half a cup', 'one quarter of a metre', or 'two tablespoons of flour to three cups of water', and so on. Teachers should use these opportunities to challenge the mathematical understandings of their students. They can do this by asking 'what if' or 'let's suppose' questions that require 'twice as much' or 'half as much'.



## Questions for reflection

How do I support my students to know the connections between fraction, proportion and percentage? Do I know these connections? If not how might I learn about them?

What is my understanding of equivalent fractions? How do I show them on number lines?

How might I teach students to find fractional quantities without using algorithms? Do I understand when fractional algorithms might be necessary for fluency following understanding?

What is my understanding of what proportions are? Am I sure? Can I reason about proportions? If not, how might I learn more about this?

Do I understand the difference between common fractions and decimal fractions? How might I teach about them highlighting connections rather than treating them as separate topics?

How do I teach students to understand proportional concepts so they do not merely learn methods and procedures for solving proportional problems?

Do my students (and I) have the conceptual understandings of proportion needed to estimate solutions first? If not how might I/we learn these concepts?

## D. Using spatial reasoning



### Key messages

There are two key ideas in the Geometry strand of the *Australian Curriculum* that are important for numeracy:

- visualise 2D shapes and 3D objects
- interpret maps and diagrams

Did you know that 'shapes' refer to 2 dimensional drawings and figures such as triangles and squares, and 'objects' refer to 3 dimensional objects such as a ball, pyramid, or cone? It is not possible to have a 3D *shape*.

In the first idea, the verb *visualise* is the key; to visualise shapes and objects you need to know about them and be able to see in your mind's eye what they look like. In order to 'see' shapes and objects it helps to think about their features. These features include lines, edges, corners and angles. In teaching and learning about shapes and objects, it helps if teachers and students have a shared language to be able to talk about them. Therefore, using the names of shapes like *square*, *circle*, *triangle* and *rectangle*, and of objects – including some mathematical objects like *pyramid*, *cone* and *sphere* – is helpful, as is building a vocabulary of the words used to describe the features of shapes and objects.

The second idea – interpreting maps and diagrams – is concerned with *where* things are rather than *what* things are. If we can *interpret* a map or diagram we can describe how to get from one place on the map to another. This means that students need to learn and use the language of direction, position and location required to do this.

### Links to the curriculum

Relevant Australian Curriculum: Mathematics Content Descriptors			
Year 7	Year 8	Year 9	Year 10
<b>Shape</b> Draw different views of prisms and solids formed from combinations of prisms			
<b>Location &amp; transformation</b> Describe translations, reflections in an axis, and rotations of multiples of $90^\circ$ on the Cartesian plane using coordinates. Identify line and rotational symmetries.			

**Relevant Australian Curriculum: Mathematics Content Descriptors**

Year 7	Year 8	Year 9	Year 10
<p><b>Geometric reasoning</b></p> <p>Identify corresponding, alternate and co-interior angles when two parallel straight lines are crossed by a transversal</p> <p>Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning</p> <p>Classify triangles according to their side and angle properties and describe quadrilaterals</p> <p>Demonstrate that the angle sum of a triangle is <math>180^\circ</math> and use this to find the angle sum of a quadrilateral</p>	<p>Define congruence of plane shapes using transformations. Develop the conditions for congruence of triangles</p> <p>Establish properties of quadrilaterals using congruent triangles and angle properties and solve related numerical problems using reasoning</p>	<p>Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar</p> <p>Solve problems using ratio and scale factors in similar figures</p>	<p>Formulate proofs involving congruent triangles and angle properties</p> <p>Apply logical reasoning including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes</p>

**Extracts from Australian Curriculum: Mathematics Achievement Standards**

<p>Students describe different views of three-dimensional objects</p> <p>They represent transformations in the Cartesian plane</p> <p>They solve simple numerical problems involving angles formed by a transversal crossing two parallel lines. Students classify triangles and quadrilaterals</p> <p>They name the types of angles formed by a transversal crossing parallel line</p>	<p>They identify conditions for the congruence of triangles and deduce the properties of quadrilaterals. Students convert between units of measurement for area and volume</p> <p>They perform calculations to determine perimeter and area of parallelograms, rhombuses and kites</p> <p>They name the features of circles and calculate the areas and circumferences of circles</p>	<p>Students explain similarity of triangles</p> <p>They recognise the connections between similarity and the trigonometric ratios</p> <p>They use Pythagoras' Theorem and trigonometry to find unknown sides of right-angled triangles</p>	<p>They recognise the relationships between parallel and perpendicular lines. Students apply deductive reasoning to proofs and numerical exercises involving plane shapes</p> <p>They use triangle and angle properties to prove congruence and similarity. Students use trigonometry to calculate unknown angles in right-angled triangles</p>
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Relevant Australian Curriculum: Mathematics Content Descriptors			
Year 7	Year 8	Year 9	Year 10
<b>Pythagoras &amp; trigonometry</b>		Investigate Pythagoras' Theorem and its application to solving simple problems involving right angled triangles  Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles  Apply trigonometry to solve right-angled triangle problems	Solve right-angled triangle problems including those involving direction and angles of elevation and depression

Australian Curriculum Numeracy Learning Continuum	
Level 5	Level 6
<b>Visualise 2D shapes and 3D objects</b> Visualise, describe and apply their understanding of the features and properties of 2D shapes and 3D objects	Visualise, describe and analyse the way shape and objects are combined and positioned in the environment for different purposes
<b>Interpret maps and diagrams</b> Create and interpret 2D and 3D maps, models and diagrams	Create and interpret maps, models and diagrams using a range of mapping tools

## Planning

Teachers and educators use the information in the 'Links to the curriculum' table in order to plan. In particular they need to understand precisely what the intended learning means. Focusing on the verbs is important and this will be described in more detail below. The verbs capture what students need to *know*, *do* and *understand* with respect to this element by the end of a particular year of the *Australian Curriculum: Mathematics* content descriptors or a specific level in the Numeracy continuum. For example, students should be able to *Develop the conditions for congruence of triangles* by the end of Year 8 if they are to achieve what is expected in

Mathematics; and *Create and interpret 2D and 3D maps, models and diagrams* by the end of Year 8 if they are to be numerate.

The activities in the following 'Putting it into practice' section describe the sorts of learning opportunities teachers need to create in order to develop the required, stated learning in the table above, both for numeracy and the mathematics learning that underpins it. The activities are not intended to be comprehensive but describe the type of activities that are required – your students might need more or less reinforcement depending on where they 'are at' in their current learning.





## Putting it into practice

### I. Visualising, describing, applying (analysing) their understanding of features and properties of 2D shapes and 3D objects (and how they are positioned in the environment for different purposes)

Notice the sequence of the verbs in the numeracy outcome for level 5: *Visualise, describe and apply understanding* of features and properties of shapes and objects. Students can visualise (see shapes in their mind's eye) based on previous learning of the nouns they use to label shapes, and can identify shapes based on their holistic recognition of these. **Visualising** is critical and was a major focus of Section D in both the K–2 and 3–6 booklets. Often teachers are unaware of the need to explicitly teach students to visualise using strategies such as: *close your eyes and picture ... picture the object as it turns around ... what can you see now?* You might also employ the draw features on your computer or software now available to students, to enable them to draw 3D objects and even turn them around on the screen.

Students should similarly be able to **describe** shapes and objects based on their *features* – edges, vertices and faces – and how the component parts of the shape fit together; they deconstruct them in their mind. Describing them depends on their knowledge of the language needed to do so, that they should have learned earlier. By secondary school, students should be able to use language of shape properties including *symmetry, angle, parallel* and *perpendicular* to describe the faces and angles of prisms and pyramids. If you show them a picture of a triangular prism for example, they should be able to describe it using terms such as *parallel faces* and *parallel edges*. If they are unable to do this you need to re-teach and encourage students to verbalise what they see. You may need to determine whether they are not doing so is because they don't know the language, or because they are too self-conscious to use it. Both these factors might need to be addressed by explicitly teaching the language and by you modelling through saying aloud and modelling what you expect them to say.

Students use their knowledge of proportion (learned simultaneously in the Number and Algebra strand and described in Section C of this resource booklet) when learning about scale factors and enlargement.

The depth of their learning of the terminology and properties of shapes and objects is evidenced in their ability to *apply* this learning to a range of familiar and unfamiliar situations. *Analysing* (level 6) takes the learning to a new level; they can compare and contrast, and deconstruct shapes and objects to determine just *how* they are combined and positioned in the environment for different purposes. Both these skills need to be explicitly taught through modelling and problem solving. You cannot assume students will be able to apply their learning or analyse situations where the learning has been applied merely because they have a knowledge of features and properties.

### Drawing prisms and objects made up of prisms

In previous years students were taught to construct prisms and pyramids; they used straws, pipe-cleaners and other materials to make faces of prisms that were the same and could be made parallel through knowing that the edges that joined the parallel faces were also parallel. This construction work provided contexts for students to learn the language to describe what they were doing and what needed to be done in order to make objects that were prisms. They learned through these activities *what the properties of prisms are*, even though they may not have formalised that understanding.

This work leads into the *drawing* of prisms in Years 7–10. Students can ensure that the front and back faces are the same (congruent) and that the vertices of the front and back identical shapes were joined by straight lines. Use construction activities (repeated if necessary) to scaffold the learning of how to draw prisms. Students should use computer 'draw' functions to ensure the front and back shapes are the same – introduce the term 'congruent' to mean 'exactly the same size and shape' since students will need this terminology later.

Have students join constructed shapes together using pipe-cleaners and straws and attempt to draw them as 3D drawings. They should learn drawing conventions and add terms such as *orthogonal* drawings (top, side, and front views), *oblique*, and *isometric* drawings to their

vocabulary. Have them draw these objects digitally and include dotted lines for edges they can't see.

You should also play 'barrier games' with students by standing at the front of the room, holding a picture of a prism, and describing what you see while they attempt to draw it on a computer. They can also replicate the 'describe what is in a picture task' in pairs so that they learn the necessary language associated with the shapes and objects. You might also give them a drawing of a prism and have them write the description of it; or get them to match a description from those generated by students to the correct object.



#### Some activities include:

- Have students work in pairs; one builds a 3D object out of cubes with some cubes completely hidden from view, and the other draws it. Have students from another group say how many cubes are in the object and how many are hidden from view.
- Barrier game (work in pairs or small groups): one student has the picture of a prism and describes it using correct geometric language; the other student/s draw it. The 'draw-er/s' compare their drawings by critiquing what each did wrong.
- Students work in groups of four: one sets up an object using cubes; the others draw it – one drawing top view, another left side view, another right side view and the other, front view. The student originally setting up the object critiques the drawings, asking the others in the group what is right/wrong with each drawing.
- Give students a piece of isometric dot paper each. Give verbal instructions for them to draw two joined sides of a prism on the dot paper (e.g. left and front) and then have them draw the remaining sides themselves. They should mark their partner's work. Then have a student stand at the front and give instructions for drawing two joined sides of a different prism with all students drawing these sides and then completing the prism.

- Repeat the above activity using square dot paper.

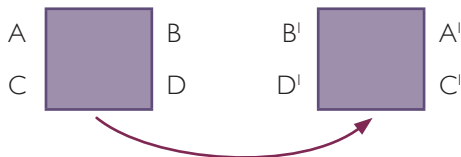
#### **Describe translations, reflections in an axis and rotations of multiples of 90 degrees**

This content descriptor, together with *Identify line and rotational symmetries* describe skills that underpin the mathematical concept of congruence. Students need to learn that when they:

- translate (or slide) a shape or object onto one the same size and shape, the original and the image are the same in every way – they are **congruent**
- reflect a shape or object by flipping it over a line/axis or turning it over onto itself through a line of symmetry, the original and the image are the same in every way – they are **congruent**
- rotate a shape or object through a number of degrees, around a point of rotation, the original and the image are the same in every way – they are **congruent**.

Students are best taught these concepts through using digital drawing tools. They can select and paste a shape or object, and then copy and paste the same shape onto another part of their page, and know that their original shape/object is congruent to the one they created by *translating* (or sliding) it; they can select and paste a shape or object, and then *rotate* it using the rotation tool, and know that their original shape/object is congruent to the one they created by rotating. Similarly, by turning their shape/object 180° over an axis or line of symmetry, they have *reflected* their shape/object and know that their original shape/object is congruent to the one they created by reflecting it in the axis (or rotating it 180°).

Whilst this explains *how* to draw these shapes, be aware that students need to be able to *describe* these transformations, using appropriate technical language as they progress through secondary school. For example, they need to learn to say: 'I reflected my square over an axis so that the original points of the corners are the same distance from the line as their images are'.



Using draw software and verbal descriptions will ensure that students are able to *visualise* what they have done, can see in their mind's eye the transformation they have used, and understand that the shapes are congruent (same size and same shape) because they have simply flipped, slid, and/or turned them.

### Geometric reasoning

Students are able to *reason* about shapes when they have a broad range of geometric language and understandings of different shapes and objects; the language to describe them and their positions; and the ability to describe what happens/results when they move. Similarly, enlarging or reducing their shapes and objects (also enabled using computer draw functions) will simplify the teaching of the concept of similarity as well as assisting the teaching of congruence.

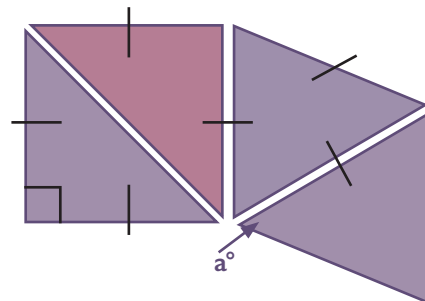
In Years 7 and 8, students learn a range of geometric properties of angles and triangles leading to properties of quadrilaterals. These include understandings about:

- Angles in a straight line, angles at a point, vertically opposite angles.
- Corresponding angles, vertically opposite angles, co-interior angles.
- Parallel lines.
- Classifying triangles as isosceles, equilateral and scalene, based on side and angle properties.
- Classifying and describing quadrilaterals.
- The sum of angles of a triangle is  $180^\circ$ ; the sum of angles of a quadrilateral is  $360^\circ$ .
- *Congruence* defined in plane shapes using transformations (described above).
- Properties of quadrilaterals using angle properties and properties of triangles.
- Properties of circles (Year 10).

These understandings require a level of *reasoning*; knowing the facts, definitions and technical language is essential but not sufficient. To really evidence understanding, students should be given questions that require them to go beyond the simple application of their learning of properties and definitions. To *reason* involves visualising: how shapes have moved, how objects have turned, yet remain congruent in spite of movement; how angles relate to one another, and in particular, how quadrilaterals are based on triangles; the building blocks of composite shapes made up of straight lines.

Students' reasoning needs to be based on these understandings. To determine that they are operating at this level of logical thought they might be given learning and assessment tasks (performed individually but preferably in pairs or groups so that they can explain and describe their reasoning out aloud) such as:

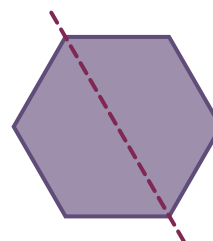
1. What is the size of  $a^\circ$ ?



2. What is one property that would make RETS a trapezium?



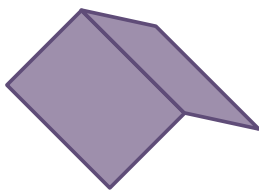
3. A hexagon is folded in half along a line of symmetry, as shown. The perimeter of half the hexagon is 20 cm. What is the perimeter of the whole hexagon?



- Draw a shape with two *internal reflex angles*. Describe the shape using correct geometric language so that a listener could draw it given your description.
- Make up a multiple choice question about quadrilaterals where the stem of the question is:

Which statement is always true? (Example: The opposite sides of a rhombus are parallel)

- A sector from a circle with diameter 18 cm has an area  $\frac{1}{6}$  of the area of the circle. What is the perimeter of the circle?
- The 2D shape shown is made up of what shapes?



(Note that while the calculation might be tedious, the reasoning is important; try to give students questions that focus on the reasoning rather than the calculation processes; digital calculating tools should always be used).

### Analysing shapes and objects in the environment and how they are combined and positioned for different purposes

The level 6 (by the end of Year 10) outcome on the Numeracy continuum *Using spatial reasoning* requires students to *analyse* shapes and objects in the environment. This is cognitively more demanding than level 5: *apply*, since it requires students to visually deconstruct shapes and objects to determine how they are constructed. Students need to see the shapes that combine to make objects and the spatial objects that combine to make the built environment.



### Some activities that give purpose to this analysis include:

- Research the requirements for the angles and lengths of ramps for wheelchairs used in building entrances. What is the relationship between their heights and lengths of ramps

and the resulting angles in the right triangles that are formed?

- Research the building requirements for widths of corridors and shower spaces in aged-care facilities.
- Examine a number of public car parks: are there any consistencies between widths and lengths of car-bays, positioning and distance from buildings, proportion of numbers of disabled car bays and width of turning spaces provided for cars reversing out of bays? Present your findings in a report for a town council and provide recommendations for improvements or increasing the number of bays.
- Investigate the place of mathematics in design of furniture, housing and public buildings.
- Explore the place of mathematics in the design of public spaces such as parks.
- Research the place of mathematics in the design of a planned city such as Canberra.
- Examine the place of mathematics in the design of arts spaces such as theatres, opera houses and galleries. For example, what is the recommended distance from which to view a painting and how high should it hang on a wall?

## 2. Create and interpret maps, models and diagrams

A map is a particular text form used to indicate location, both of *where* things are positioned in their environments and *where* they are positioned with respect to other shapes and objects within the same environment.

At level 5 (by the end of Year 8) students use grids and coordinates to represent locations and direction; they use the Cartesian plane and use pairs of numbers to describe where something is located. Whereas in algebra they plot ordered pairs using the points of intersections of grid lines, in geography (maps using latitude and longitude, and including street maps) they locate things using numbers that define the square *within* which an object is located. The numbers are often located *between* the lines rather than *on* the lines. Students will need to be taught this difference explicitly.

By Year 8, students should be able to use technical terms and measurements associated with compass directions (and later bearings), degrees of turn, distance, scale and coordinates. The concepts need to be understood if students are to *create and interpret* maps as required. Be aware that if students can't understand or haven't grasped these concepts, the roots of misunderstanding often lie in the fact that they have not grasped other mathematics ideas. For example, students who don't understand scale on a map are often those who struggle with fractions, measurements and proportion. Some of the activities presented in the 3–6 resource booklet should support students in re-visiting these concepts to learn and/or consolidate them.

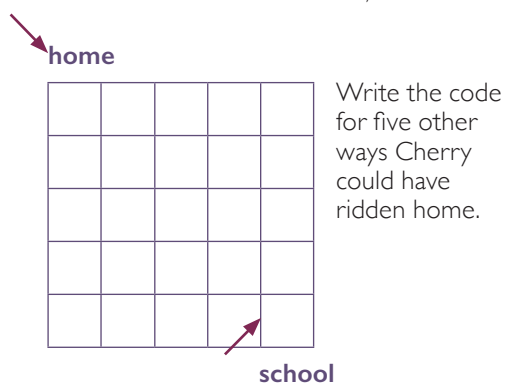
Many students who don't understand scale – used to determine distances between places on maps – don't understand proportion. For example, to know that 'on the map 1 cm is equivalent to 100 cm on the ground' and that this is represented by the symbol 1:100 requires students to understand the symbolic representation and how to interpret it proportionally. These skills need to be explicitly taught.



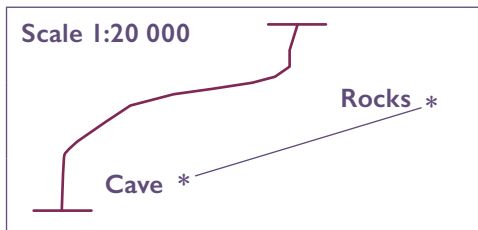
### Some activities that support students to learn to create and interpret maps

- Students work in pairs and take turns; one finds a place of interest on a map from an atlas and gives the latitude and longitude coordinates for it to the other who has to find the place on another map which is the same.
- Students work in pairs to develop a PowerPoint presentation that presents the difference between coordinates in the plane and grid references on a geographical map such as a street directory.
- One student wears a cap with a country written in block letters sticking up from the cap (the wearer does not see the country's name). The student asks questions of the rest of the class requiring yes/no answers from the audience such as: is it in the Northern hemisphere? Is it between 0°N and 33°N? The winner uses the least number of questions to locate the place.

- Barrier game: one student uses a street directory, giving the starting point and describing directions to another place using correct language including distance, clockwise and anti-clockwise, right-left, degrees of turn, and so on, while the other draws a 'mud map' of the given directions. At the end they compare maps and see how close the 'drawer' is to the location described.
- Students program a vacuum cleaner robot giving it directions to clean the inside of the school buildings, including moving from room to room. Students work in pairs and then compare another pair's instruction to see if they can understand the path of the robot.
- Students work in pairs to develop a map of the school that can be given to new students and visitors. The map should be to scale and include compass points.
- Students make up questions about directions that they could be asked by people in the street e.g. Where is the Post Office? Where is the Information Centre? Where is the Recreation Ground? They must write directions for how to get there from where they are, including estimating distances, accurate turns, number of streets on left and right to be passed, roundabouts, and so on.
- James was facing north-east and then turned 180° to his right. Which direction was he then facing? Students write similar questions for each other.
- Students work in pairs to create a fun run event and are told that the route must be exactly 10 km long.
- Cherry uses a grid (as shown) to record her ride home from school as 4N, 4W

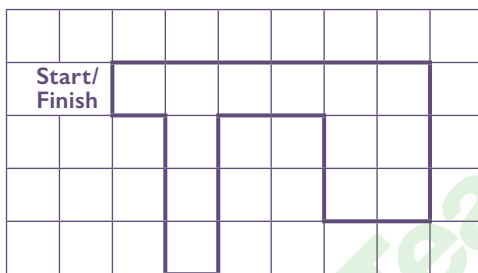


- On this map the distance from the cave to the rocks is 3.5 cm.



What is the actual distance from the cave to the rocks on the ground?

- Christy walks along a path as shown on the map (grid squares are 100 m by 100 m)



How far did Christy walk in kilometres?

Students might think that *exact maps* are not needed anymore due to digital tools such as GPS. However, they need to know how to determine whether instructions are appropriate or not. For example, an instruction such as: *Go south-west and turn right* requires a user to know these directions even though the location point is shown on the digital tool. From a safety perspective, the visual tool should not be solely relied on. Similarly, sometimes a GPS might give the shortest route which can go through bushland and terrain requiring a four-wheel drive vehicle. Or, it might take a truck through a CBD area if programmed for the *quickest* route. Users need to make decisions about what maths to use in these contexts; they need to know *where they are* and have a sense of which direction they are going in, in the same way that they should estimate numbers, costs, distances and measurement attributes in other contexts.

*Digital tools are accurate but they are also very literal and need to be critiqued in the same way that all digital tools do.*

At level 6 the focus is on more sophisticated maps such as those that show arrangements relating to travel rather than exact distances. These include train maps that show the 'next station on the line'. Exact distances are not needed since the rider needs to know only where to get on and where to get off. Whilst all maps do this, network maps such as those used on trains, focus on what lies *between* two points. At level 6 students should also learn about bearings used for navigation.

Creating these maps and diagrams efficiently, sometimes involves using a range of digital mapping tools such as Google Earth, MapLib, NeatLine, WorldMap, StoryMaps, SketchUp, and online Tasmanian resources such as those from DPIPWE (see references) which can help develop map understandings and visualisation. Teachers should experiment with these tools and consider how they might help students, particularly in applying their mathematics learning to real life contexts where map drawing and connection to other texts are needed.



#### Some activities that support students to learn to create and interpret more sophisticated maps

- Students investigate an orienteering map to see how distances and directions are shown.
- Students work in pairs to develop an orienteering/rogaining map that can be used by the local orienteering and/or rogaining club.
- Students use network maps (such as a map of the London or Singapore underground system) to plan routes from one location to another given conditions such as not going through the same station twice; taking the shortest route, and so on.
- Students work in pairs to create a network map of their local community featuring major locations such as the school, post office, cemetery, in one line.
- Students work in pairs to create a network map of their local region featuring local towns and tourist locations and showing the relationships between them.





## Monitoring and assessment

All the above tasks and problems can be used for assessment as well as teaching. Students can be asked to individually find solutions and to draw and write about why they know they are correct. You might also give them a question with a hypothetical student's response, and have them mark it: *Are they right? How do you know? How might you support them if they have made an error?*

A good source of assessment questions for students in Years 7–9 is some of the questions in past Year 7 and 9 NAPLAN Numeracy test papers. The questions generally align closely with the definition of numeracy used in this resource and:

- provide no hints about the mathematics students should choose to use
- are for the most part written in contexts so that students have to understand these before choosing the mathematics to use.

## Links across the curriculum

Other curriculum areas provide contexts for the application of mathematics. Teachers need to draw the links by asking students: *Will some maths help here? Or, what maths are we using here?*

Some of these contexts **demand** understanding of the mathematics concepts in order to understand them. For example, students won't understand some objects unless they are able to describe them using spatial language. You can either show students how useful their maths learning can be in these contexts or use the contexts as **opportunities** for *teaching* the mathematics concepts.

There are some numeracy links across the curriculum identified on the *Australian Curriculum* website:

In **History**, students use maps to understand migration patterns, including urbanisation and the drift to cities as a result of droughts and other natural phenomena. Maps also support understanding of historical events such as the transatlantic slave trade and transportation of convicts and the roles these played in social

reform and in developing the United States and Australia.

In **Geography**, students might study maps showing the starting points of peoples on the move when studying social justice issues of internationally displaced peoples due to ethnic conflicts. These require maps showing positions, distances travelled, key features in migration routes such as deserts and oceans, and compass directions. Teachers should use these as opportunities for students to see the value of maps in real-life situations and historical contexts, and impress on them the importance of being able to read maps in order to understand issues of international importance. Maps also support the study of international trade and issues of distance and proximity to markets for example when studying the feasibility of developing Australia's north. When supporting students to read authentic maps, ensure they understand how scales work: most standard *TasMaps* are 1:100 000 (so 1 cm on the page = 100 000 cm or 1 km). Spent Maps at: <http://dpi.wa.gov.au/land-tasmania/spatial-discovery/listmap-in-schools> support both history and geography.

In **English**, students might use simple grids to determine positions in stage layout and actor positions. They might also come across instructions for camera positions in filming that require camera angles. Some of the texts they study might include simple maps to help them understand the narrative e.g. a map of The Shire in *The Hobbit*. These are opportunities to support students in understanding how to read and interpret scale, legends and positions of places relative to other places.

In **Science**, their study of plate tectonics in Year 9 will be better understood through the visualisation of continental movement over time. Using local urban maps in Science is a also good way of introducing displacement as opposed to distance in Year 10 – even for simple speed and distance calculations.

In **Languages**, students learn about the countries where their target language is spoken. Specifically teaching about places on a map using the vocabulary often requires students to understand scale, compass points and directions. This is an opportunity for Languages teachers to reinforce and apply the mathematics of map reading in a real context.



## Questions for reflection

How do I plan experiences which support my students to know how to visualise something?  
How do I know when they can visualise?

Have I explicitly taught my students how to visualise something? Have I modelled this?

How do I support students to draw what they see 'in their mind's eye'?

Are my students able to draw shapes and objects from a written or verbal description?  
Can they provide descriptions of shapes and objects using mathematical language to describe features? If not, what do I need to plan for and explicitly teach them?

Can they describe an object using technical language appropriate for their stage of learning?  
What words and phrases do I need to model and focus on? What words and phrases might I need to learn?

How well do my students understand transformations: reflection, rotation and translation and the practical applications of these for concepts such as symmetry, clockwise and anti-clockwise turns, and drawing parallel faces in prisms?

Can students recognise and name prisms and pyramids in the environment? If not, how do I build this understanding?

Do my students follow directions given in language that includes compass points, scale, turns, and left and right to go somewhere? Can they give simple directions using the same sort of language to explain to someone how to get somewhere? What can I model, demonstrate or use to support them in building this understanding?

What experiences do I give my students to use maps? How do I know how well they do this?

## E. Interpreting statistical information



### Key messages

There are two key ideas in the Statistics and Probability strand of the *Australian Curriculum* that are important for numeracy:

- interpret data displays
- interpret chance events.

Data that is collected, organised and displayed by someone else is called **secondary** data; if we collect, organise and display it ourselves it is called **primary** data.

In the first idea, interpret data displays, the verb *interpret* is the key; to interpret data displays does not mean you have to be able to collect and organise data or draw the display. Whilst it helps if you know a little bit about collecting, organising and displaying data, these skills are not essential

for interpreting it. However, students who do these things have to consider **purpose and audience**: *whom are we doing this for and why do they want to know?*

The second idea – interpreting chance events – is concerned with interpreting information about the *likelihood* of something happening.

Words such as *fifty-fifty* or '*fat chance*' occur in most students' homes. The use of these words is usually based on long-term experience in our lives. Sometimes this might include statistical analysis that produces a number that is used as a **measure of chance** and this is what **probability** focuses on.

However, to have a sense of risk involved in taking an action does not need deep knowledge of probability or analysis; experience is sufficient. In a court of law for example, a jury might convict on the basis of 'beyond reasonable doubt' which means there is a relatively small chance that they are wrong. In attempting to measure the possibility of an event occurring, there is always a risk.

Students need to be taught what phrases like *slight chance* mean so that they can make informed decisions when taking risks. The complexity and depth of understanding increases as we have greater experience with risk in our lives. If we learn about this from a mathematical perspective we will be better informed later in life when we need to make decisions about how much risk we are prepared to take, in financial planning for example.

## Links to the curriculum

Relevant Australian Curriculum: Mathematics Content Descriptors			
Year 7	Year 8	Year 9	Year 10
<p><b>Data representation and interpretation</b></p> <p>Identify and investigate issues involving numerical data collected from primary and secondary sources</p> <p>Construct and compare a range of data displays including stem and leaf plots and dot plots</p> <p>Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data</p> <p>Describe and interpret data displays using median, mean and range</p>	<p>Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processing</p> <p>Investigate the effect of individual data values, including outliers, on the mean and median</p> <p>Explore the variation of means and proportions of random samples drawn from the same population</p> <p>Investigate techniques for collecting data, including census sampling and observation</p>	<p>Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources</p> <p>Construct back-to-back stem and leaf plots and histograms and describe data, using terms including <i>skewed</i>, <i>symmetric</i> and <i>bi-modal</i></p> <p>Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread</p> <p>Investigate techniques for collecting data, including census, sampling and observations</p>	<p>Determine quartiles and inter-quartile range</p> <p>Construct and interpret box-plots and use them to compare data sets</p> <p>Compare shapes of box plots to corresponding histograms and dot plots</p> <p>Use scatter plots to investigate and comment on relationships between two numerical variables</p> <p>Investigate and describe bivariate numerical data where the independent variable is time</p> <p>Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data</p>
<p><b>Chance</b></p> <p>Construct sample spaces for single-step experiments with equally-likely outcomes</p> <p>Assign probabilities to the outcomes of events and determine probabilities for events</p>	<p>Identify complementary events and use the sum of probabilities to solve problems</p> <p>Describe events using language of <i>at least</i>, <i>exclusive or</i> (A or B but not both), <i>inclusive or</i> (A or B or both) <i>and</i></p> <p>Represent events in two-way tables and Venn Diagrams and solve related problems</p>	<p>List all outcomes for two-step chance experiments both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events</p> <p>Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'</p> <p>Investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate populations means and medians</p>	<p>Describe the results of two –and three – step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events</p> <p>Investigate the concept of independence</p> <p>Use the language of 'If...then', 'given', 'of', 'knowing that', to investigate conditional statements and identify common mistakes in interpreting such language</p>

Relevant Australian Curriculum: Mathematics Content Descriptors			
Year 7	Year 8	Year 9	Year 10
<b>Extracts from Australian Curriculum: Mathematics Achievement Standards</b>			
<p>Students identify issues involving the collection of continuous data</p> <p>They describe the relationship between the median and mean in data displays</p> <p>They construct stem-and-leaf plots and dot-plots</p>	<p>Students model authentic situations with two-way tables and Venn diagrams</p> <p>They choose appropriate language to describe events and experiments</p> <p>They explain issues related to the collection of data and the effect of outliers on means and medians in that data. Students determine complementary events and calculate the sum of probabilities</p>	<p>Students compare techniques for collecting data in primary and secondary sources</p> <p>They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes</p> <p>They construct histograms and back-to-back stem-and-leaf plots</p>	<p>They compare data sets by referring to the shapes of the various data displays</p> <p>They describe bivariate data where the independent variable is time. Students describe statistical relationships between two continuous variables</p> <p>They evaluate statistical reports</p> <p>Students list outcomes for multi-step chance experiments and assign probabilities for these experiments</p> <p>They calculate quartiles and inter-quartile ranges</p>

Australian Curriculum Numeracy Learning Continuum	
Level 5	Level 6
<p><b>Interpret data displays</b> Compare, interpret and assess the effectiveness of different data displays of the same information</p>	<p>Evaluate media statistics and trends by linking claims to data displays, statistics and representative data</p>
<p><b>Interpret chance events</b> Describe and explain why the actual results of chance events are not always the same as expected results</p>	<p>Explain the likelihood of multiple events occurring together by giving examples of situations when they might happen</p>

## Planning

Teachers and educators use the information in the 'Links to curriculum' table in order to plan. In particular they need to understand precisely what the intended learning means. Focusing on the verbs is important and this will be described in more detail below. The verbs capture what students need to *know, do and understand* with respect to this element by the end of the particular year of the *Australian Curriculum* or specific level in the Numeracy continuum. For example, students should be able

*to identify complementary events and use the sum of probabilities to solve problems* by the end of Year 8 if they are to achieve what is expected in Mathematics; and *Compare, interpret and assess the effectiveness of different data displays of the same information* by the end of Year 8 if they are to be numerate.

The activities in the following 'Putting it into practice' section describe the sorts of learning opportunities teachers need to create in order to develop the required, stated learning, both for numeracy and the mathematics that

underpins it. The activities are not intended to be a comprehensive list but describe the type of activities that are required – your students might need more or less reinforcement depending on where they 'are at' in their current learning.



## Putting it into practice

### Interpreting data displays 7–10

Interpreting data is the culmination of collecting, organising and displaying data – it is the *purpose* for engaging in these activities. Technology can be increasingly programmed to collect, organise and display or represent data, but it is difficult to program it to effectively *interpret* the data. Note that **you cannot learn to interpret data by only engaging in the skills of collecting, organising and displaying it alone**. People don't learn what graphs mean by drawing them, for example. This isn't to say that the skills of developing and asking the right questions, collecting and organising data from asking the questions, and displaying the data in a range of formats, is not important. For numeracy, the skill of reading and interpreting graphs *that someone else has drawn* is the most important skill for the majority of the population. Our lives are surrounded by data which has often been interpreted by someone else for their particular purpose; and sometimes that is to influence you! Teachers should focus on supporting their students to understand the context (purpose and audience) of a graph to assist critical interpretation, as a major focus for numeracy.

It is important that we address this balance in classrooms; for many years – prior to the availability of technologies that digitally draw graphs – we focused on the displaying of data and drawing graphs and charts. When technology made it possible to display a chart by pressing a key on a keyboard, this impacted significantly on the teaching of graphs in mathematics classes. Teaching students in classrooms to overly focus on drawing graphs may be perceived as 'busy work' since they are learning skills that are needed less. We should instead give greater focus on teaching students to *interpret* data displays since these skills are increasingly important in society.

In Years 7 and 8 (**level 5 Numeracy**) the focus has shifted from that in earlier years from collecting and arranging data in various ways, to comparing the effectiveness of different displays for different purposes. In Years 9 and 10 (**level 6 Numeracy**) this focus is further transformed to a higher cognitive level where students are expected to be able to evaluate the effectiveness of data displays and critique their effectiveness for a given purpose. Given that many displays are media-based it is clear that the intention of the display might not only be to provide information but may also be to achieve a political effect. Students need to be aware of this and to look for ways that the media may have manipulated the display to bias the interpretation.

In Years 7 and 8 it is important to use data that students can relate to; primary data that they collect themselves is a good starting point and teachers gradually extend this to working with data from secondary sources. You might also use contexts that engage students in discussion about social issues such as obesity, smoking, relationships, pollution and forestry practices. As students mature, their focus becomes broader and they can engage with issues such as domestic violence, reconciliation and climate change.

Increasingly, contexts will be presented through daily media in newspapers, online, and web-based research, and teachers should draw students' attention to the number of news items that use data and statistics to support their case and even persuade readers/viewers. Increasingly students will realise the need to become data-literate in order to be informed citizens; being able to understand links between stories in the media and the data used as part of media texts is an essential skill for people in the twenty-first century.

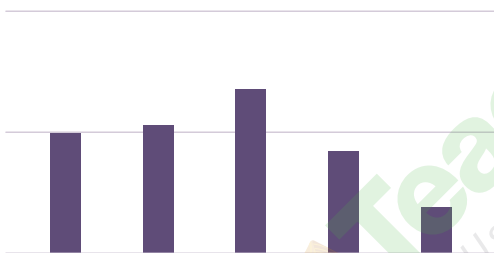




### Some activities for interpreting data in graphs

Critical skills for interpreting data in graphs can be found embedded in the following questions; teaching should focus on the learning and application of these skills by students for numeracy acquisition.

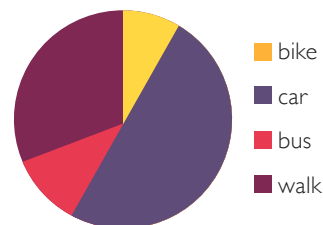
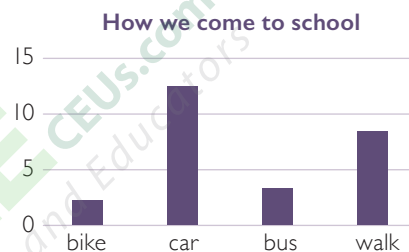
- Give students a graph such as the one shown, having no indication of what is on the axes, just a title: *People who play sport*. Have students break into groups and discuss what the axes might represent, and what the numbers on the axes might be. They might need to do some research to find out.



- Students might then speculate on what sample was used, how the data was collected, who are the people, and so on, depending on what their research reveals or doesn't. They might make up some questions of their own to ask a sample of students to see if they can 'mirror' the data in some way to help them decide which people were surveyed in the first instance.
- Ask students: 'which way do most of us come to school?' Have them form small groups or pairs and discuss how they might find out. They will need to consider the question they will ask, the recording sheet they will use, and how they might display their data. In displaying their data, their table might be something like:

Bike	2
Car	13
Bus	3
Walk	8

- In deciding how to best display their data they should enter it onto a spread sheet and then think about the question: Do we really need to know how many people come to school using different ways or do we just need to be able to look at the graph quickly to know the answer? They might decide that exact numbers are not needed to find how 'most of us' come to school, so a circle graph is sufficient – we don't need any scales marked on axes.

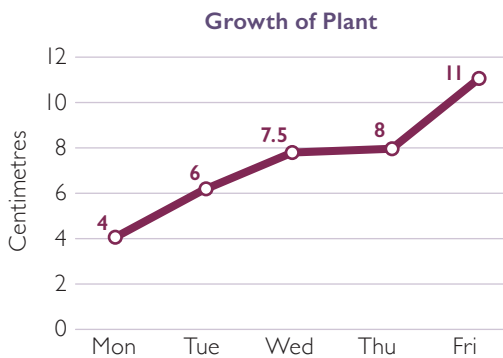


- Students need to know that some data *vary over time*. They might best learn this when taking measurements in science activities such as measuring the growth of a plant or measuring the temperature every two hours. Whenever they plot data points that can be joined up (i.e. the line joining the data points *means* something) this is the case.

They might collect data such as:

Day	Mon	Tue	Wed	Thu	Fri
Height of plant	4 cm	6 cm	7.5 cm	8 cm	11 cm

By putting this data on a spreadsheet they will get a display such as:



Students can see that the plant is constantly growing during the week so the lines joining the data points are important; it wouldn't make sense to use a column graph.

Ask students what their graph is showing them:

- How much did the plant grow between Monday and Friday?
- Between which two days did it grow the most?
- Which day didn't it grow much compared to the other days? Explain how you know this?

Many middle-years students have misconceptions about 'average'. Their understandings about the concept are mostly median-based, to a lesser extent mode-based but rarely mean-based. Teachers need to ensure that they teach the concepts associated with measures of central tendency, rather than just rules and procedures for finding each of the measures. Students need to know what they are finding when they calculate the mean. If they understand what it is they can estimate it based on the raw data.

- Provide students with a range of statements from the web that refer to the *average* in different contexts. For example, *the average length of a bed in Russia is 1.8 metres; the average life span of a whale is 80 years*. Have them work in pairs to discuss whether the mean, median or mode would have been used for *average* in each case and have them justify their reasons for this.

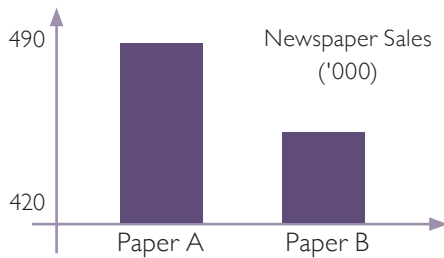
- Gather some graphs from the newspapers and have students work in groups to consider what questions might have been asked; who might have been asked; over what period of time people were asked; whether outliers might have been removed from the data and why; and any other interesting information.
- Show some graphs (or tables) that include obvious outliers. Ask students how this might have happened and consider possible errors in data collection and how they occur. Consider also, that 'obvious outliers' may in fact be legitimate and should not be dismissed too easily.
- Year 7/8 classes at MC College collected bottles and jars for recycling. They weighted their bottles and jars and put the information in a table, as shown:

Class	June	July
7A	5.6 kg	7.21 kg
7B	3.24 kg	3.8 kg
7C	3.14 kg	6.99 kg

Which type of graph would show these results in the way that best indicates which class collected the greatest mass of glass? Use a spreadsheet and plot your graphs using different charts to help you decide and justify your choice.

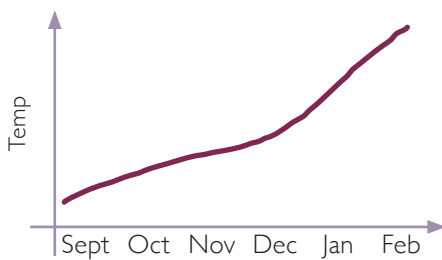
- Misleading graphs: Many publications display charts that are deliberately misleading in order to try and influence their readers, especially in advertising, or advertorials. See if you can find some of these yourself (including on the web by searching for 'Misleading Graphs') or make up some of your own for students to 'find the biases'. For charts from real sources, have students work together in pairs, to not only find the bias, but also consider the possible reasons for publishing the misleading graph.

a. Missing sections of an axis:



b. Incomplete data:

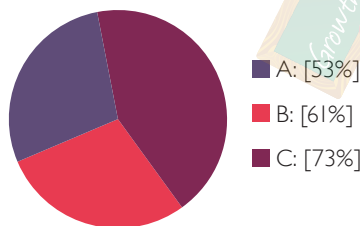
The case for global warming: It's Getting hotter!



c. Figures that don't add up!

Pre-election polling indicates voters favour politician C.

**Politicians Favoured to Win Election**



The following table shows the number of people at a barbeque:

	Females	Males
<b>Children</b>	35	20
<b>Adults</b>	90	75

- What fraction of the adults at the barbeque were males?
- What proportion of the females at the barbeque were adults?

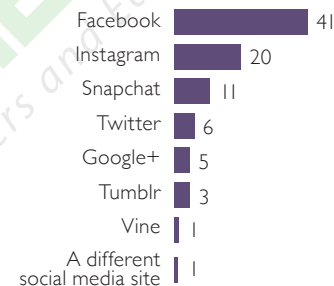
• Graphs from the State newspaper:

- Students evaluate (put a value on) statistics they find in the media in tables and charts. They should particularly consider charts that extrapolate the data and extend trends, asking whether the trends are legitimate and the level of risk involved in making predictions about trends in world stock markets, for example.

• Consider the following graph and answer the questions:

**Facebook, Instagram and Snapchat Used Most Often by American Teens**

% of all teens who use \_\_\_\_\_ most often



Note: \*Don't use any responses not shown.

Source: Pew Research Center's Teens Relationships Survey, Sept. 25-Oct 9, 2014 and Feb. 10 Mar. 16, 2015 (n=1,060#teens ages 13 to 17).

PEW RESEARCH CENTER

- What social media do most teenagers in the US currently use?
  - What does '41' represent?
  - What ages are the 'teens'?
  - How big was the sample of teens used?
- Lead a discussion about which graphs are better for displaying data about proportions. If we are not really concerned with exact numbers but just proportions (i.e. not absolute values but relative ones) what sort of graph best shows this type of data? A bar graph? Histogram? Line graph? You might give them some data showing population sub-types that make up your city or state and have them do a presentation on which type of graph is the most useful for comparing the population sub-groups? When is a line graph useful? What sort of graph best suits time-series data, and so on? You might have to match types of data sets with the most appropriate chart; they should justify their answers using technical language appropriate to their level of learning.

## Interpreting chance events 7–10

It is important for teachers to know that chance isn't about probability. In fact, it is the other way around; probability is about chance. Probability is about measuring chance or **measuring likelihood**. Most people in their everyday lives aren't concerned with *measuring* the likelihood of something happening or not. They are satisfied to have a general sense about likelihood – this is demonstrated in the language we use when talking about chance.

Some students develop misconceptions about chance through playing with dice or tossing coins. If they always seem to get a head when tossing a coin they might believe that they are more likely to get this every time. If they often get a number that *isn't a 6* when they throw a die they might believe that it is harder to get a 6 than any other number. They might even believe it is easier to get a 3 or a 4 when throwing a die since that's what they always get! These misconceptions can prevail even into adulthood with some adults believing that:

- after a run of bad luck your luck will eventually change
- an event is more likely if it happens to someone you know (this is why some newsagents publicise the fact that they sold the winning ticket).

If students learn that the result of one throw with a die or a coin is not influenced by the previous throw they are less likely to develop this misconception or to continue to have it as they mature. They need to learn that some events are **independent** of each other. Teachers can help students overcome these misconceptions by asking questions that prompt students to reflect on their beliefs. For example they might ask: *When throwing a die is it more likely I'll get a four than a six? Who thinks so? Why do you think that Jimmy? Why don't you think that Sam?* Promoting classroom discussions and debate can lead students to readjust their beliefs and help overcome these misconceptions.

They also need to learn that some events are **dependent** on each other, that is, the result of one experiment can sometimes affect the result of the next experiment. For example, if you draw out a card from a standard pack and get the queen of hearts and don't put it back, you can't get the queen of hearts on the next draw; the result of your second draw **depends** on the result of your first.

You should teach students that we can use *numbers* as a measure of chance; that '50% chance' means it is equally likely to happen or not happen. Students should learn that most of these figures come from undertaking an experiment many, many times so that we can say 'in the long run' the chance of either outcome is equally likely. They learn that by tossing a coin ten times it is unlikely that they will get exactly five heads and five tails. If someone else in the class tosses a coin ten times they are very likely to get different results – this variation in results is normal and should be expected. However, if they toss a coin a million times they are more likely to get *about* 500 000 of each but still very unlikely to get *exactly* 500 000 of each. The concept of variation is an important one for students to learn; if they expect a result when they take a chance they will usually be disappointed.

**Chance is about risk and there is no such thing as a 'sure thing' if you can't control the outcome.**

It is critically important that students explore these issues and misconceptions related to chance as they move into adulthood. Young people might hold false beliefs about risk in relation to vital issues such as:

- The chance of getting cancer is ...
- The chance of dying of lung cancer if you smoke are ...
- The chance of your child getting complications from immunisation are ...
- The chance of having a car accident if you speed is ...
- The chance of winning lotto if you buy a ticket every week is ...
- The chance of making lots of money if you invest in X is ...

Understanding the true level of risk is key to making constructive life decisions as adolescents and later adults.



### Some activities for interpreting chance events

Teaching should focus on students learning and applying these skills for numeracy acquisition. Skills for interpreting chance events can be developed through activities and questions such as:

- Have students get in pairs, each with a coin. They should toss their coin 100 times and determine how many heads and tails they get. Record the results for each pair in the class. Discuss the fact that there are some with more than fifty heads and some with less than fifty heads, so *in the long run* they are more likely to have about 50 of each.
- Students repeat the first activity using numerical representations (percentage, decimal or fraction) for each of the likelihoods e.g. '1' for certain. They can predict values for those that are unlikely or likely.
- Students make their own spinners so that they have  $\frac{1}{4}$  chance of stopping on blue,  $\frac{1}{8}$  chance of stopping on red,  $\frac{3}{8}$  chance of stopping on black, and  $\frac{1}{4}$  chance of stopping on green. (These fractional amounts can be altered for another activity).
- Students discuss the fact that on the weather on TV last night the presenter said: 'there will be 10% chance of rain'. How might the weather bureau arrive at that figure?
- Students discuss the publicised fact that there was 1 in a 3,838,380 chance of winning first division in the recent lotto in Australia. What does that mean? Are they more likely to win than someone else? What if they've bought a lotto ticket every week for three years?
- Jaimie had a bag containing 15 different pieces of fruit. The bag contained equal numbers of oranges, apples and nectarines. He reached in and got a piece of fruit without looking. What is the chance that the piece of fruit was an orange or a nectarine?
- A standard die is rolled once. What is the probability that the number on the top face of the die is a) a multiple of 2? b) a factor of 4?
- Your neighbour has just won \$40 000 in lotto and your Dad is convinced that he has a good chance of winning lotto also. Write a paragraph advising him not to 'waste his money', giving sound reasons. Compare what you have written with a friend's version.
- A journalist wrote an article in the paper explaining that because 'one in 100 people win money when betting on horses' then this means that two in two hundred people win money betting on horses. He argues that since the last 198 people through the gates have not won any money, 'the next two people through the gates will win'. Your task is to write a letter to his editor explaining why he is wrong, giving sound mathematical reasons.
- Eighty percent of smokers across Australia wish they had never started smoking. How do you think this figure was determined? Would researchers have interviewed all smokers in Australia? If you start smoking, what are the chances that you will wish you never had sometime in the future? Why?
- A headline in a newspaper advertisement stated: 'Average people make their fortune' with a by-line of: 'Invest your life savings in *PropertyRich* and live your dream'. What advice might you give your parents or grandparents if you find they are considering doing just that?



## Monitoring and assessment

Assessing the interpretation of data 7–10 involves giving students lots of practice with different types of displays and contexts that involve chance, including tables and charts. Students need to be explicitly taught some strategies for doing this in step-by-step and holistic ways, since a data display and statements involving chance can be very complex texts for students – and adults – to read.

The above tasks can all be used for assessment as well as for teaching and learning. Even though you have used some contexts for teaching you can always adjust the tasks for assessment. The focus should be on students interpreting the data rather than collecting and organising it. Whilst you can give students interpretive questions to assess how well they can read tables and charts, you might also consider giving them tables and charts and have *them* write the questions. You should ensure that any questions you write cover the whole range of thinking from directly reading values, to evaluating changes, and projecting beyond the data given, especially to consider possible trends. They can predict what might happen if ...? They should also consider the long-term data trends by looking at the bigger picture and 'smoothing' the data to get rid of any highs and lows that might occur. This can also be done for data collected from chance processes, where risks have been taken.

For scenarios involving chance, you should ensure that students have the opportunities to consider risks and interpret them in terms of the implications for making judgements about risks such as in financial planning, addictive behaviours and their own health.

A good source of assessment questions for students in Years 7–9 is some of the questions in past Year 7 and 9 NAPLAN Numeracy test papers. The questions generally align closely with the definition of numeracy used in this resource and:

- provide no hints about the mathematics students should choose to use
- are for the most part written in contexts so that students have to understand these before choosing the mathematics to use

## Links across the curriculum

Other curriculum areas provide contexts for the application of mathematics. Teachers need to draw the links by asking students: *Will some maths help here? Or, what maths are we using here?*

Some of these contexts **demand** understanding of the mathematics concepts in order to understand them. Others are **opportunities** for teaching the mathematics concepts.

There are some numeracy links across the curriculum identified on the *Australian Curriculum* website.

In **Science**, students are learning inquiry skills which require them to generate, collect and organise data from their own investigations and experiments. You would encourage them to draw a graph of this data and make predictions or draw conclusions as part of the laboratory process. They would summarise data from their own investigations and secondary sources. They would also evaluate the quality of data collected and identify improvements in data quality to ensure accuracy appropriate to the task.

In **History**, students are learning population proportions of different migrant groups and considering other statistics based on population movement, urbanisation, production and trade. These contexts provide ideal situations for reading and interpreting data displays of various types. In considering the slave trade and convict transportation there is opportunity for students to interpret data shown in different displays and to consider the historical impact of different migrations on Australia and other parts of the world.



In **English**, you might encourage students to support their arguments with data and graphs when writing a persuasive piece. You should show students in Years 7 and 8 how complex a chart is as a text type, and teach them how you read a chart by viewing it holistically first, reading the title and axes headings. You could then show them how to 'read' a data point, asking: 'what is it telling you?' They will need to consider what it represents on the one axis and then the other axis.



## Questions for reflection

How do I help my students understand that different charts are used to display data for different purposes and audiences? That they should decide how to display the data based on the question it is answering? Do I understand this? If not, how might I learn more about it?

How well do I understand what variation in data is about? How might I learn more about this concept if I need to?

Do I know how to read graphs and interpret them? What strategies do I use to interpret graphs? How do I share and model my understandings with students?

How do I understand the words of chance like observation, likelihood, event and experiment? How confident am I to teach these words accurately?

How might I support students to understand the difference between dependent and independent events?

Do I understand how important it is for adults to understand what risk is about, and that while risk can be managed and minimised, it can't be controlled?

Might I have misconceptions about chance that I could unwittingly pass onto my students? How confident am I about that? How might my colleagues support me to know more about this concept?

## F. Using measurement



### Key messages

There are two key ideas in the Measurement and Geometry strand of the *Australian Curriculum: Mathematics* that are essential for Numeracy:

- estimate and measure with metric units
- time: operate with clocks, calendars and timetables

Both of these ideas are about Measurement rather than Geometry.

**Estimate and measure** are the two verbs used in the first idea and their order is important; there is no point estimating *after* you have measured. If the **context** you are working in demands an accurate measure, you will estimate first to give you some idea of what measurement to expect. That is, your estimation will give you confidence that you have measured correctly when you *do* measure.

Sometimes estimation is all that is required in the context in which you are working. If you don't need to be very accurate and you can tolerate some error, an estimation might be sufficient. For example, if you want to buy a new wardrobe for your bedroom, you might use a hand span to decide how wide it will need to be to fit all your clothes in or how narrow it needs to be to fit through your bedroom door. However, if you live in a remote location and need to phone in an order for the wardrobe because you can't go to the city to buy one, you need to measure the width *accurately* using metric units since you can't use a hand span as a unit of measure when ordering because it is not accurate enough.

As with calculation, **it is impossible to estimate without deep knowledge of what you are estimating**. So, if you don't have a deep understanding of the attributes length, area, volume, capacity *and* the units used to measure them – in this case, metric units – you will not be able to estimate their measurements. To estimate measurements you need to see *them*

*in your mind's eye* and use known measures as benchmarks. For example, *I know what a metre 'looks like' so I can look at the length of that rope and use my 'visualised metre' to estimate how long the rope is.*

To measure requires some deep understandings about:

- What needs to be measured (understanding **attributes**).
- **Units** of measurements (so you can estimate and choose an appropriate tool to measure with).
- **Estimating**, using your understanding of attributes and units.
- **Measuring**, using **direct measure** (doing the measuring yourself) and **indirect measure** (using a combination of measures or measures given to you) methods.

Working with and understanding **time** is an essential part of Western civilisation. Our society demands that we can read measures of time (including clocks, calendars and timetables) and can determine elapsed time. Our days, months and years are measurements in units of time that are unusual because it is slightly different from the other attributes of length, area, mass and volume/capacity, and angle; we can't see or heft it – instead we *experience its passing*

## Links to the curriculum

Relevant Australian Curriculum: Mathematics Content Descriptors			
Year 7	Year 8	Year 9	Year 10
<p><b>Using units of measurement</b></p> <p>Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving</p> <p>Calculate volumes of rectangular prisms</p>	<p>Choose appropriate units of measurements for area and volume and convert from one unit to another</p> <p>Find perimeters and areas of parallelograms, rhombuses and kites</p> <p>Investigate the relationship between features of circles such as circumference area, radius and diameter. Use formulas to solve problems involving circumference and area</p> <p>Develop the formulas for volumes of rectangular prisms and prisms in general. Use formulas to solve problems involving volume</p>	<p>Calculate the areas of composite shapes</p> <p>Calculate the surface area and volume of cylinders and solve related problems</p> <p>Solve problems involving the surface area and volume of right prisms</p>	<p>Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids</p>
<p><b>Time</b></p> <p>Tell time to the minute and investigate the relationship between units of time</p>	<p>Solve problems involving duration, including using 12 – and 24-hour time within a single time zone</p>	<p>Investigate very small and very large timescales and intervals</p>	
Extracts from Australian Curriculum: Mathematics Achievement Standards			
<p>Students use formulas for the area and perimeter of rectangles and calculate volumes of rectangular prisms</p>	<p>Students solve problems relating to the volume of prisms</p> <p>They make sense of time duration in real applications</p> <p>Students convert between units of measurement for area and volume</p> <p>They perform calculations to determine perimeter and area of parallelograms, rhombuses and kites</p> <p>They name the features of circles and calculate the areas and circumferences of circles</p>	<p>Students calculate areas of shapes and the volume and surface area of right prisms and cylinders</p>	<p>Students solve surface area and volume problems relating to composite solids</p>

Australian Curriculum Numeracy Learning Continuum	
Level 5	Level 6
<p><b>Estimate and measure with metric units</b> Convert between common metric units for volume and capacity and use perimeter, area and volume formulas to solve authentic problems</p>	Solve complex problems involving surface area and volume of prisms and cylinders and composite solids
<p><b>Operate with clocks, calendars and timetables</b> Use 12 – and 24-hour systems within a single time-zone to solve time problems, and place personal and family events on an extended time scale</p>	Use 12 – and 24-hour systems within a multiple time zone to solve time problems, use large and small timescales in complex contexts and place historical and scientific events on an extended timescale

## Planning

Teachers and educators use the information in the 'Links to the curriculum' table in order to plan. In particular they need to understand precisely what the intended learning means. Focusing on the verbs is important and this will be described in more detail below. The verbs capture what students need to *know, do and understand* with respect to this element by the end of the year of the *Australian Curriculum* or level in the Numeracy continuum in which it is positioned. For example, students should be able to *Find perimeters and areas of parallelograms, rhombuses and kites* by the end of Year 8 if they are to achieve what is expected in Mathematics; and *Convert between common metric units for volume and capacity and use perimeter, area and volume formulas to solve authentic problems* by the end of Year 8 if they are to be numerate.

The activities in the following 'Putting it into practice' section describe the sorts of explicit learning opportunities teachers need to create in order to develop the required, stated learning in the table above, both for numeracy and the mathematics learning that underpins it. The activities are not intended to be a comprehensive list but describe the type of activities that are required – your students might need more or less reinforcement and depending on where they 'are at' in their current learning.



## Putting it into practice

### I. Understanding attributes

Part of being numerate in measurement is understanding *what* needs to be measured in a context. It is therefore important that students understand measurement attributes. **If we want to measure something we are interested in how much of a particular attribute it has.**

An **attribute** is a quality that an object has. It might have length or area, mass or volume/capacity, or angle, and it will often have more than one attribute. We can compare objects by their attributes (e.g. *it has more mass or it is longer*) and sometimes different attributes might result in different orders when we compare (e.g. one object might be *taller* than another but that same object might be *lighter* than the other)<sup>xiv</sup>. Whereas we can estimate the length by looking at the object it is often difficult to estimate the weight by looking; we need to heft it.

Once we have decided what to measure we need to think about the purpose or *why* we want to measure it. The purpose helps us decide the **level of accuracy** or **precision** that is needed. Sometimes it is not helpful to be more accurate; an estimate is sufficient for the purpose and accurately measuring just takes too much time. For example, if we want to paint a room we don't need to work out the surface area of the walls; it is enough to know that a four-litre tin of paint is usually enough to paint a room that size with a couple of coats of paint.

Most of the ideas about attributes were introduced in the K–2 years. However, there are some additional qualities that should be defined in greater detail for the 7–10 years, being aware that many students who didn't grasp these understandings earlier really need to know them NOW before they can develop their understandings of the intended learning for Years 7–10.

### What is the difference between volume and capacity?

Capacity is how much something will hold. Volume is the space taken up by whatever is filling the object, or the space taken up by the object itself.

### What is the difference between area and perimeter?

Most of us have no problems at all understanding perimeter as the distance around the edge of a shape. So we say 'the shape has perimeter'. Perimeter is measured in length; one attribute is measured by another attribute, so it is measured indirectly.

The concept of area is more challenging. Many teachers find *area* difficult to define. As a result, many students define area as 'length times width'.

**Area** is the amount of surface an object or shape has. Area is a quality of a surface just as perimeter is a quality of a shape. Any surface has area – it does not have to be a flat surface but can be a curved surface or a jagged surface. Area is measured *indirectly* since we need to use a *square* unit to measure with: it has equivalent length and width.

## 2. Understanding units

It is impossible to estimate measures without understanding the units that we use to measure with. Although the element of the Numeracy continuum is *Estimate and measure with metric units*, young students need to estimate and measure with **non-standard units** before they can do these things with **metric units**.

Measuring requires us to work out how many **repeats of the unit** we use are needed to match the thing to be measured. This is essentially what measuring is about. So the understandings needed to measure using units include:

- some objects work better than others as units to measure with (due to gaps and overlaps)
- the bigger the unit chosen to measure with, the smaller the number of repeats
- for comparison you need to use the same unit
- the purpose for measuring tells us which unit will give the most accuracy (and how much care is needed in measuring)
- standard units are no more correct than non-standard units; it depends on the context.

As students mature they are able to work with standard units as a more accurate way of communicating about measures; it is more consistent to use formal standard units since they are the same for everyone and hence do not vary. Whereas hand spans, lolly wrappers and cups can vary considerably depending on those we use, units such as metres, centimetres and litres are *standard*; they are exactly the same size wherever we are in the world. As a result, it is better to use standard units than non-standard units if we want greater accuracy and if we want to communicate the measures to someone else.

It is helpful if we choose to measure with a unit that relates well to the attribute we are measuring. For example, we wouldn't use a ruler to measure *mass*, even though a ruler would be a good measuring unit if we were measuring *length*.

### The importance of visualisation

Understanding the units we use is helped if we can **visualise** the units i.e. have a picture of them in our head. Visualisation is essential for estimating both direct and indirect measures. Students need to be able to 'see' metric units and use these mental images as benchmarks. If we can 'see' a square metre we can look at a floor rug and estimate how many of these could fit into the rug – hence our estimation is based on the mental image we have of the square metre, our benchmark.

## We need to teach students what these mental images look like.

Some activities to build students' mental images:

### Millimetre:

- Draw a millimetre on each student's paper. Have them write: 'This line is one millimetre long'. Don't use a ruler to show them this length; they will think the measure 'belongs on a ruler'. Next have them draw a millimetre in length on their page 10 – 20 times. Ask them if they can think of anything that is about one millimetre long; make a list of things on the board. Send them outside in pairs to make a list of anything they can see outside that is a millimetre long. Come back together and share.

### Square millimetre:

- Have students draw a square millimetre on their page; it should be a square which is one millimetre long and one millimetre wide. You may need to show them this yourself but don't enlarge it to draw on the board. Draw it on each student's page if you have to. Ask students if they can think of anything that has a surface of one square millimetre. They can go outside in pairs and look as well. Share, on coming back to class.

### Cubic millimetre:

You might want to leave this one until after you have done a square centimetre, then do the same activity as described for the square centimetre, below.

### Centimetre:

- Do the same as the millimetre task above. Practise these two measures by having students draw a millimetre and a centimetre each day on their page. Have them close their eyes and picture a millimetre and then a centimetre.

### Square centimetre:

- Explain to students that a square centimetre is a square that is one centimetre long and one centimetre wide. See if they can draw it. (Don't draw an enlarged one on the board). If you have to draw one on each student's page or have someone who drew it correctly help you to draw it on other students' pages. Have students write: 'This is

a surface of one square centimetre' on their page. Ask students to think about what thing they know that has a surface of about one square centimetre. Have them work in pairs to create a list of things with surfaces that big. Have them shut their eyes and picture a square centimetre and see if they can draw it on their page with their eyes shut! Generate a list of things on the board with a surface of about one square centimetre. Give students some square grid paper and have them colour in a square centimetre. Ask: 'How many square millimetres do you think would fit into a square centimetre?' See if they can draw them. You should continue asking them to draw centimetres and millimetres and square centimetres and square millimetres each day. Extend the activity by asking them to draw six different shapes with a surface area of four square centimetres. Talk to students about whether their versions are correct, or if not, why they don't meet the criteria.

### Cubic centimetre:

- Explain to students that a cubic centimetre is a box drawn on a square centimetre so they have a cube which stands on a square centimetre but is also one centimetre tall or high. Show them a cubic centimetre (use a wooden cube). Have them try to draw one; this might be tricky but at least they should be able to visualise one when they shut their eyes. See if they can think of other things in their environment that are the same size. Use these cubes to build objects that have no more than six cubic centimetres. Students then share their objects, seeing how many they can come up with as a class. Talk about how the objects should have at least two faces together i.e. they can't only be joined at a corner but must be joined along a face. Practise visualising one cubic centimetre (cc), two ccs, three ccs, and so on, every day so students can visualise this measure clearly and describe it: 'it is a cube with length, height and width one centimetre long'.

### Metre:

- Show students what a metre looks like – don't use a ruler! Outstretch your arms and say: *The length from the tip of my hand to the tip of my other hand is about one metre long. Does anyone think their outstretched arms are one metre long? Come and compare with*



mine and we'll see whether your outstretched arms are smaller or bigger than mine. Is there anything else in the room that might be one metre long?

Square metre:

- Ask students what a square metre might be (based on their knowledge of a square centimetre and square millimetre). See if they can tell you – perhaps with some prompting – that it is a square with a side one metre long and one metre wide. Take them outside and give them some chalk. See if they can draw a square metre on a paved area. Alternatively you can have them stand on the corners of a square metre which you have drawn and look at the shape they create on the ground. Have them take a good look at this size. They can then work in pairs to use some newspaper to create a square centimetre of their own using sticky tape or glue. Have them use their square metre model to estimate the size of the handball court by repeating its use over and over until the court has been covered. Another activity is to use their model of one m<sup>2</sup> and see how many square metres other surfaces in the school yard might be e.g. rose garden, verandah, basketball court, under-cover area, and so on.

Cubic metre:

- Make a model of a cubic metre in the class (or if you are lucky enough to find a box this size you can have it on display). You can use this as your benchmark: 'Is this cupboard/box/storeroom bigger or smaller than a cubic metre?'

Ensure that all students have a mental picture of what metric units look like and their size. You can then begin to ask them to estimate using these sizes as benchmarks e.g. *How long is it from the floor to the window sill? Is it more or less than a metre? How long is the room? Is it more or less than five metres? What is the surface area of your desk? Is it more or less than a square metre? What is the surface area of this book? Is it more or less than ten square centimetres? Look at this cupboard; will it hold more or less than one cubic metre? Look at this shoebox; will it hold more or less than 20 cubic centimetres?*

Hectare:

- Although a hectare is not a common unit for students (or even some adults!), it is the most common unit used in real estate so naturally we would want students to know how big it is. Take students out to the school oval and borrow a hundred metre tape or trundle-wheel. Measure (or pace) one hundred metres straight, do a quarter turn and then measure another hundred metres. Repeat this until you are back where you started and have students stand on each corner. All students should take a good look at this hundred-metre square since this is a hectare! This benchmark can be used for estimating the size of blocks of land. Students should be able to see that a school oval is about 2.5 hectares, the school grounds are about 10 hectares and the local park is about 5 hectares, and so on.

Kilometre:

- Ask students to tell you how long a kilometre is – you may be shocked at some of the answers! Take your class for a walk down the road, one kilometre exactly (make sure you've measured this in your car first). Then turn around and come back to class. By the end of the period they will know what a kilometre 'looks like'.

Right angle:

- To estimate angle sizes students need only know 90°. They will have learned in the study of transformations that 90° is a *quarter turn*. They know that 90° is a square corner and that this can be used as a benchmark to estimate angle sizes; half a quarter turn is 45° so a quarter turn plus half a quarter turn is 135°.

### 3. Estimating measures

The link between understanding attributes and units, and estimating, becomes clear when you consider what is needed in order to estimate. Clearly you can't estimate how much of an attribute something has if you don't understand the attribute or the units used to measure it. For example, how can you estimate the area of a carpet if you don't understand what *area* is and if you don't have a mental picture of a unit used to measure it, such as a square metre?

**Estimation** is an 'informed guess'. It is making an approximation based on the information available to you; what you can see, heft or experience (in the case of time). Estimation is helpful when it is difficult to measure something directly.

Estimation depends on **purpose and audience**. It is these two criteria that help you decide whether 'near enough is good enough'. We should always ask: 'are we confident that our estimate is good enough in the circumstances or should we calculate with greater precision?'

In the example given about estimating the quantity of paint needed (in 1. above) the question about how much error we can tolerate in estimating the number of four-litre tins of paint we need to paint the walls of our room, will depend on how far we live from the shop – if we are wrong and need to buy more it is a significant issue if we live 600 km from the shop compared with if we live just around the corner from it. So whereas we might be able to tolerate an over-estimate we may not want to risk an under-estimate. We need to have a sense of **confidence in our estimate** in order to minimise the risk of being too far wrong.

The confidence in our estimate will improve over time if we practise **visualising units**.

### 4. Measuring (direct and indirect)

**Direct measuring** requires us to work out **how many repeats of the unit we use are needed to match the thing to be measured**; we do this ourselves rather than using a formula or measurements that someone else has made.

Initial measuring tasks involve counting units and we need to be careful that our students learn that measuring and counting are not the same thing. Measuring is used for **continuous** quantities. Whilst we can count how many students are in the class or how many cats are in the garden as **discrete** quantities, quantities such

as length, area and volume cannot be counted since they are continuous quantities. **The measurement of any continuous quantity can only be approximate, not exact.** If we say that a book is twelve centimetres wide what we are really saying is that it is about 12 centimetres wide; it is closer in width to 12 cm than to 11 cm or it is between 11.5 cm and 12.5 cm. The more accurate we want to be, the greater precision is needed in our act of measuring and in the size of units marked on our measuring instrument.

Students should learn to measure using 'between' or 'to the nearest' statements<sup>xv</sup>. They wouldn't say 'this jug holds six cups of water' but rather 'this cup holds between five and seven cups of water' or this jug holds six cups of water *to the nearest cup*. **The smaller the unit used to measure with, the greater the accuracy.** In using a cup there will always be some spillage or error since you are only using your eye to 'fill' the cup. In using a 30 cm ruler marked with millimetres they are more likely to reduce the measurement error when measuring the length of their pencil than if they used a whiteboard ruler marked with centimetres.

Students need to learn what the appropriate measuring tool is to measure each attribute and to use the tool in ways that reduce measurement error. They should complete tables such as:

Attribute	Suitable units	Suitable measuring tool
Length of pencil		
Mass of an egg		
Volume of a box		
Capacity of a cup		
Area of a book cover		
Height of a tree		
Area of a rug		
Width of a garden bed		

Students should have many opportunities to talk about which unit is best for measuring each of these attributes and why or why not (in terms of gaps and overlaps or ability to move). If young students have not learned these concepts in K–2 they need to be re-taught or at least revisited in Years 3–6. Be aware that some students in Year 7 might still have not developed an understanding of these ideas.

**Indirect measuring** also requires us to work out **how many repeats of the unit we use are needed to match the thing to be measured**, but we use it when the object or shape is large or complex. In finding the area of a large park for example, it would be too laborious or time-consuming to use a model of a square metre and lay it out over the park repeatedly to find how many square metres of ground surface in its area. When this is the case we might use a formula made up of a number of different *measures* (e.g.  $\text{area} = \text{length} \times \text{width}$ ). It would be simpler to directly measure the park by driving along the border in a car, determining how many metres long and wide it is, and working out the area indirectly by *using the direct measures* in this formula.

In learning to measure, young students will not (and should not) use formulae. They will, however, use indirect measuring by **combining** direct measures or to **compare** and **order** attributes. For example, they might compare the lengths of a range of objects they can move by placing them alongside each other. If the objects are too large or fixed in position they can compare by using a third, movable object, as a **go-between**. For instance, we can compare the heights of doors by using a broom as my 'matching unit'.



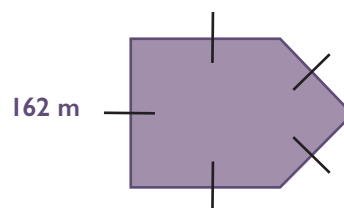
#### Some activities for estimating and measuring with metric units

- Often students struggle to measure lengths with a piece of a ruler – they always want to start the measurement from the end marked with a '0'. Give them a partial ruler (or piece of paper with ruler markings) and have them discuss how they might use it to measure the lengths of objects in their room such as whiteboard height from floor, length of teacher's desk, height of the window sill etc.

Students should estimate each measurement before calculating.

- Which is the longest distance: 2460 cm, 24060 mm, 0.2406 km or 246 m?
- Students examine a thermometer with positive calibrations only and beginning at zero. Where does the liquid start – at the 'bowl' or the '0' mark? Have them research and then write instructions for someone else on how to read a thermometer to tell the temperature. They should give some examples of different thermometers which have different scales marked (including those that have negative calibrations) and where the liquid comes up to different points along the scale.
- Students are given the task of determining how much juice to purchase for a class function. They will need to consider how much each person will drink (what 'cup' will we use to decide this and how much does it hold?); how many cups of juice will we need? How large is the container that we can buy juice in? How much does it hold? How many will we need? They can work as a class or in pairs and present their work as a report showing their answers clearly and supported by reasons.
- Students have to research the difference between area of a rectangle and perimeter of a rectangle. They should develop a five-minute *PowerPoint* presentation, using the 'border' and 'shade' functions to clearly indicate the difference, and different types of rectangles to illustrate how these amounts can change; what happens to the perimeter and the area if the rectangle is doubled in length? Halved in length? What happens if it is twice as long and half as wide?
- Students work in pairs to investigate the relationship between surface area and volume; what happens to the volume if the surface area is doubled? Trebled? Halved? They should construct a table of values and then graph the relationship in order to describe it.
- Jemma bought a floor rug with an area of 1.75 square metres. Will it fit on the floor of her bedroom which is 1.35 m long and 1.45 m wide? If not, how much bigger or smaller is the rug?

- Investigate the relationship between the area of a circle and its diameter. Construct a table of values to help you.
- Use cubes to develop the formula for the volume of a rectangular prism. Build the prism up layer by layer, calculating the volume of each layer. When you think you understand the relationship between the volume of one layer and the number of layers in the prism, test this with another rectangle as the base, and then another. Can you generalise the relationship?
- Measure the amount of water dripped by a tap in a twenty-four hour period. How much would be dripped during a week? What about a year?
- Examine the relationship between the areas of a parallelogram and a rectangle with the same length?
- Alain made a cake that weighs 456 grams. Steve's cake weighs 0.47 kilograms. Whose cake weighs more and by how much?
- Draw a thermometer that is showing a temperature of  $37^{\circ}\text{C}$ . Indicate gradations every five degrees.
- Use a computer-generated grid (or grid paper) to draw six different triangles, each with an area of 24 square centimetres. Is there a relationship between their heights and the lengths of their bases?
- Four toddlers were weighed on the same day at the clinic. Simon weighed 6.50kg, Sarah weighed 6450g, Meredith weighed 6.04kg and Ozzie weighed 6082 grams. Which toddler has the greatest mass?
- Draw a garden of an unusual shape which has an area of 6.8 square metres.
- Franci makes a punch for her party. She uses at least one full bottle of each of the following bottles of juice: orange juice ... one bottle contains 500 ml, lemon juice ... one bottle contains 200 ml, apple juice ... one bottle contains 300 ml ... and guava juice ... one bottle contains 600 ml. How many full bottles of juice will Franci have to buy to make 2L of punch?
- A garden has an area of 24 square metres. What might its length and width be? How many different answers are there? Explain your thinking.
- Justin is an apprentice bricklayer. How many bricks will he need to construct a brick wall five metres long and three metres high if the length of a row of 18 bricks is 4.2 metres long? Draw a diagram and estimate your answer before you start.
- Gemima bought a fish aquarium at the pet shop. It had a volume of a half of a cubic metre. Will this fit in the back of her (standard) empty ute? What might its dimensions be?
- Donald bought a cubic metre of mulch. If he spreads it on his front lawn at a depth of ten centimetres, how much lawn can he cover?
- A Madagascar monkey can jump up to five times its body length. If a Madagascar monkey is 75 cm long, how far can it jump, in metres?
- A standard car trailer holds about a cubic metre of sand. If a trailer is half full, how many cubic centimetres will it hold?
- If a car trailer is 1.8 m long, 1.4 m wide and 40 cm deep, will it hold a cubic metre of mulch? How many trailer-loads will the owner need to make to collect 1.5 cubic metres of mulch to cover his lawn?
- Two sides of a triangle measure 6 cm and 14 cm. Write some number sentences that describe what you know about the length of the third side.
- A square has an area of 145 square metres. What is its perimeter?
- The area of a rectangle is 36 square metres. What might its perimeter be?
- Find the surface area and volume of a cylinder with a radius of 12 cm and height of 1.8 metres.
- A farmer wants to build a fence around his paddock (below) with 4 strands of wire. How many kilometres of wire are needed?



- The length of a rectangle is half its height and its perimeter is 30 cm. What is its area?
- What is the surface area of a tennis ball with radius of 5 cm? It is hollow when cut in half but you can see that its wall is 0.5 cm thick. What is its capacity?
- A house has a roof in the shape of a rectangle 6 m by 8 m. If 2 centimetres of rain falls on the roof and flows into a tank alongside the house, how much rainwater will be collected?

### Operate with clocks, calendars and timetables

As with other attributes, time is measured by counting units. Students need to understand the units of time just as they need to understand the units they can see or feel. Whilst students can 'have a feel' for small units of time such as seconds, minutes, hours and days, units larger than that – weeks, months or years for example, may be beyond their comprehension depending on their age and experience.

Increasingly, students are operating in an international environment, making friends with people all over Australia and the world, many of whom live in different time zones. Young people need to consider what the time will be in other places when they make contact; as they get older and make contact with businesses and work colleagues, they need to calculate 'what will the time be there?' As with most measurement estimating, they should consider: *How much error can I tolerate?* They need to know for example, that if they are calling their Mother in the middle of the night from the other side of the world they can generally tolerate more error than if they were calling their employer.


These calculations can be done mentally and frequently, using estimation, by knowing a number of 'benchmarks'. In Australia for example, students should know Eastern, Western and Central Standard Time. They should use and be familiar with these at different times of the year, adding or subtracting an extra hour or hour for 'summer time'. Internationally, they might know a few times for key cities around the world. For example, they know that London is ten hours behind Melbourne and that Milan is eight hours behind Melbourne, which is three hours behind Perth. These benchmarks help them to estimate the current times in places like Mumbai, Cape Town and New York.



### Some activities for operating with clocks, calendars, timetables (and extended time scales)

Have two clocks on the wall: one analogue and one digital. Refer to them during the day, asking questions about when lesson changes will occur.

- Jasmine fell asleep at 7.45 pm and woke up at 6.45 am. How long did she sleep?
- 2016 is the fourth leap year in the twenty-first century. What is the 20th leap year?
- Give students tables to complete with different times, clocks and words:

Time in words	Time on digital clock	Time on analogue clock	Different words for the same time
Quarter to five in the morning	04:45		Four forty-five

Have students mark these in pairs and correct each other's work when needed.

- Have students make up multiple-choice questions about their day and put them all together on a worksheet for the class to do (they should also write their answers down and give these to the teacher to use for marking). For example: Sheila went to the tennis court at 11.15 and returned at 12.02. How long was she at the tennis court? Sian went to the library at 9.35 and returned to class in 1 hour and 23 minutes. When did she return?
- Give students a list of times taken for each of fifty people in a fun run, written in digital format e.g. 20.07 minutes. Have students work together in pairs to put them in order to find the first ten placeholders.

- Download sections of bus timetables for your local area and give them to students, having them work in pairs to write questions about time taken and finishing times. The following table uses WA place names:

9.45	Esplanade Station
9.46	Canning Bridge Station
9.52	Murdoch Station
9.57	Bull Creek Station
10.03	Cockburn Central Station
10.10	Kwinana Station
10.13	Wellard Station
10.21	Warnbro Station
10.35	Mandurah Station

- How long does it take the bus to go from Murdoch to Warnbro?
  - Is it faster to go from Bull Creek to Wellard than from Canning Bridge to Kwinana?
  - The express train from Esplanade to Mandurah takes 19 minutes. How long will it take to go from Esplanade to Mandurah stopping at all stations? How much time will you save by catching the express train?
- When it is 12 noon in Adelaide it is 10.30 am in Bangkok on the same day. A plane leaves Adelaide at 9 am Adelaide time and flies to Bangkok. The flight takes five hours. What time will the plane arrive in Bangkok?
  - The sun rises at 6.34 am and sets at 7.13 pm. How many hours of sunlight is this?
  - If the sun sets at 19:23 and we have 14 hours of sunlight, what time on the 24-hour clock does it rise?

- Mitch plans to leave Albany at 7.30 am to drive to Perth through Mandurah. He finds this table of driving times:

Albany to Mandurah	4 hours 20 minutes
Mandurah to Perth	55 minutes

If Mitch stops at Mandurah for a 35-minute break, what time should he arrive in Perth?

- John lives in Perth. He wants to phone Mrs Adams when she starts work at 8.30 am in Tasmania in summer. What time should he phone her?
- A movie has a playing time of 127 minutes. If Karen starts it playing the movie at 9.04 pm and has no breaks, what time will it end?
- Students in history were learning about Ancient Greece. They learned that Pythagoras was born in 571 BC and Aristotle was born in 384 BC. Who was born first of these two and by how many years? How many years ago was Aristotle born if it is 2015 now?
- Fred phones Elise in Milan at 2 pm where it is 8 am in the morning on the same day. She wants to call him back at about 7 pm so that Fred will be home from work. What time should she phone him?
- The disk in the hard drive of Sam's computer makes 85 full turns every second. How many minutes will it take to make 120 000 full turns?
- Cosi is researching his family tree. He knows his great grandfather died at the age of 79 in 1932. What year was his great grandfather born?





## Monitoring and assessment

In order to *monitor* the learning against what students are expected to learn, teachers need to refer back to the intended learning in the *Australian Curriculum*. They need to continue to remind themselves what their particular goals are.

A good source of numeracy assessment questions for students in Years 7–9 is some of the questions in past Year 7 and 9 NAPLAN Numeracy test papers. The questions generally align closely with the definition of numeracy used in this resource and:

- provide no hints about the mathematics students should choose to use
- are for the most part written in contexts so that students have to understand these before choosing the mathematics to use.

One approach might be to select particular questions related to the current teaching focus and project them onto a whiteboard. The teacher can then discuss the various strategies which might be used to solve the questions and also address particular misconceptions students appear to have about the multiple choice options. *Why might someone select A? What would someone be thinking if they thought the correct answer was C?*

## Links across the curriculum

In the increasing specialisation of secondary teaching areas, including option or elective areas such as the Technologies subjects, the importance of numeracy demands and opportunities takes on a richer meaning for teachers. Who better to demonstrate the applications of mathematics in real-life contexts than the teachers of these subjects? Teachers need to draw the links by asking students: *Will some maths help here? Or, what maths are we using here?*

Some of these contexts **demand** understanding of the mathematics concepts in order to understand them. For example, students won't understand some types of change unless they are able to measure the differences. Similarly they won't understand timelines unless they can put

numbers in order. You can either show students how useful their maths learning can be in these contexts or use the contexts as **opportunities** for *teaching* the mathematics concepts.

In **Foods and Hospitality**, measurement is essential. Discussions should include the concept of *how much error can we tolerate?* So purpose and audience are necessary considerations. If you are cooking at home for your family for example, *exact* measurement may not be critical. However, if you are preparing a dinner party for friends, the tolerance of error increases somewhat, and increases again if you are preparing a meal for an important occasion for which you will receive payment.

Some contexts for using measurement in this subject area include:

- Converting spoon or cup measures in some recipes to millilitres
- Estimating cutting widths e.g. each slice should be 2 cm
- Rolling dough e.g. roll dough until it is 1.5 cm thick
- Measuring quantities using measuring cups; should the flour/sugar be exactly at the line? Just above/just below/straight? What difference might it make?

In **Languages**, students learn new vocabulary in cultural contexts such as cooking. They would use a recipe in the target language and learn different units of measurement. They might engage in measuring the heights of students, and in measuring distances. Teachers should use this as an opportunity to support their students in measuring tasks since the skill of measuring is the same in any language. Choosing a tape measure and making sure the end of the measure is at the very end of the object being measured; measuring liquid amounts in cups and jugs, and measuring mass by weighing are all legitimate Languages tasks where students are applying their mathematics learning using another language.

In **Design and Technology, (Wood and Metal)** a range of measuring tools is used to measure length, angle, mass, and volume including:

- Rulers (to set the length of a drill bit to acquire a certain depth of a hole), Vernier

gauge 0.001 mm (to check thickness of materials), digital callipers 0.01 mm, 5 mm protractor, sliding bevel (to calculate mitre cuts), scales 0.01 g, beakers and measuring cylinders, and set square.

- Teachers instruct students to set machinery by appropriately calculating, measuring and checking to minimise waste; if recalculation is required, students measure again and adjust, checking again, and so on.
- Students must measure temperatures; heating or cooling to correct temperature for mixing chemicals or when bending materials such as acrylic.

A great deal of this work requires accuracy in measurement, both in the design and construction phases. In design and planning, material wastage must be considered when calculating cost, and in construction incorrect measuring and checking can mean mistakes and waste.

Similarly, in **Visual Arts**, students must calculate costs of materials such as paper and canvas size based on measuring materials required: *Will the canvas fit in the space on the wall? Weights of clay and plaster must be considered in designing pieces: Will they fit on the shelf? Will they be too heavy to carry? What length of picture wire do I need to hand my canvas? What gauge will it need to hold the weight?*

There are some other numeracy links across the curriculum identified on the *Australian Curriculum* website:

The **Science** learning area uses a variety of instrumentation for measuring a wide range of units. Teachers should take these opportunities to ask students which instrument is most appropriate for measuring units in context, and asking why, rather than making these decisions themselves. They might even assess student understandings of attributes and measurement in context by asking which units might be measured with particular instruments.

In **Science**, when students are learning inquiry skills they might use contexts that generate data over time e.g. growth of plants. They would use metric units to measure how much a plant has grown overnight, or sugar and water solutions to compare dissolving rates. As the science teacher you would involve students in the decisions about which tool to use to measure with to get the level of accuracy required, and how to graph their measurements to best display their data for purpose and audience. You would point out to them that: 'we are using some maths here', or you might use such a lesson to teach the concepts of all using the same unit so they can compare whose plant has grown the most. You should engage them in discussions about error tolerance: 'How much error can we tolerate here?' 'What difference will it make to our results if we don't measure the amount accurately?'

In **History**, students can apply their knowledge of time by sequencing objects and events by placing historical events in order by date from most recent to least recent. They can also use their measurement knowledge in determining and comparing the length of voyages and treks between and across continents when learning about explorers.

In **English**, students might measure surfaces on a stage for drama to determine the area needed for a production; allow and encourage them to decide which units of measurement to use and which tools to use for the measurements – or ask: *Are estimates enough?*



## Questions for reflection

How well have I taught students to visualise standard metric units of length, area and volume?

How well are my students able to estimate the number of metric units in a shape or object by looking at it?

Do my students understand what the attributes of perimeter and area are? How do I know? How might I support those who do not?

Are my students able to compare how much of an attribute a shape or object has, and use the appropriate language to explain why? What explicit teaching do I need to plan if they cannot?

Do my students know what is and what isn't an appropriate unit to measure an attribute, and can they tell me why?

Do my students understand the difference between area and perimeter? Between volume and surface area? Between volume and capacity? What learning experiences will help them know this?

Are my students able to estimate lengths, areas, perimeters and volumes by visualising metric units and comparing? Can they look at a trailer for example, and estimate how much it can hold?

How well do my students tell the time on analogue and digital clocks? Can they estimate times in other time zones? How might I model and plan to support them?

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# APPENDIX I: USEFUL RESOURCES

*The following are examples of resources available to support numeracy development.*

## Key national and state documents

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Tasmania's Literacy and Numeracy Framework 2015–2017 <https://www.education.tas.gov.au/documentcentre/Documents/Tasmanias-Literacy-and-Numeracy-Framework-2015-2017.pdf>

Supporting Literacy and Numeracy Success [www.education.tas.gov.au/documentcentre/Documents/Supporting-Literacy-and-Numeracy-Success.pdf](http://www.education.tas.gov.au/documentcentre/Documents/Supporting-Literacy-and-Numeracy-Success.pdf) (staff only)

Good Teaching: Differentiated Classroom Practice [www.education.tas.gov.au/documentcentre/Documents/Good-Teaching-Differentiated-Classroom-Practice-Learning-for-All.pdf](http://www.education.tas.gov.au/documentcentre/Documents/Good-Teaching-Differentiated-Classroom-Practice-Learning-for-All.pdf) (staff only)

Good Teaching: Quality Assessment Practices [www.education.tas.gov.au/documentcentre/Documents/Good-Teaching-Quality-Assessment-Practices-Guiding-Learning.pdf](http://www.education.tas.gov.au/documentcentre/Documents/Good-Teaching-Quality-Assessment-Practices-Guiding-Learning.pdf) (staff only)

Good Teaching: Curriculum Mapping and Planning [www.education.tas.gov.au/documentcentre/Documents/Good-Teaching-Curriculum-Mapping-and-Planning.pdf](http://www.education.tas.gov.au/documentcentre/Documents/Good-Teaching-Curriculum-Mapping-and-Planning.pdf) (staff only)

The Early Years Learning Framework for Australia [www.dss.gov.au/sites/default/files/documents/05\\_2015\\_belonging\\_being\\_and\\_becoming\\_the\\_early\\_years\\_learning\\_framework\\_for\\_australia.pdf](http://www.dss.gov.au/sites/default/files/documents/05_2015_belonging_being_and_becoming_the_early_years_learning_framework_for_australia.pdf)

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Australian Curriculum Numeracy General Capability [www.australiancurriculum.edu.au/generalcapabilities/numeracy/introduction/introduction](http://www.australiancurriculum.edu.au/generalcapabilities/numeracy/introduction/introduction)

NAPLAN Toolkit <https://login.education.tas.gov.au/login.aspx?appID=41&redirect=http%3a%2f%2fnaplan.education.tas.gov.au%2fdefault.aspx> (staff only)

National School Improvement Tool [www.acer.edu.au/files/NSIT.pdf](http://www.acer.edu.au/files/NSIT.pdf)

## Whole School Approach

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*Department of Education Improvement Plan – Improving Student Achievement through a Whole-School Approach* <https://www.education.tas.gov.au/documentcentre/Documents/Improving-Student-Achievement-Through-a-Whole-School-Approach.pdf> (staff only)

## Numeracy assessment

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### Assessment for Common Misunderstandings

(Victorian Department of Education)

[www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/pages/misunderstandings.aspx](http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/pages/misunderstandings.aspx)

One on one assessment of key ideas in number (development of other strands under way).

Freely available from this site and through Scootle. Highly diagnostic with teacher advice rubrics on where to go next to target teaching.

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### IMPROVE

[www.improve.edu.au](http://www.improve.edu.au)

Access past NAPLAN questions and develop quizzes and tests for formative assessment. Free to teachers.

Logging in: Teachers in Tasmanian Government schools log in using their DoE username and password.

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### Scaffolding Numeracy in the Middle Years

<http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/pages/scaffoldnum.aspx>

Provides access to a developmental continuum focusing on key ideas in middle school numeracy. Assessment tools are included along with advice for targeted teaching.

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## Teaching practices

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**AITSL:** information and videos on quality numeracy practices.

Information and videos on quality numeracy practices.

<http://www.aitsl.edu.au/australian-professional-standards-for-teachers/illustrations-of-practice/detail?id=IOP00070>

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### TCH: The Teaching Channel

An excellent collection of videos demonstrating effective classroom strategies for numeracy teaching. It has been developed to support the US national curriculum, but has many videos relevant to Australian schools.

<https://www.teachingchannel.org/>

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## Numeracy resources

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### Scootle

Scootle.edu.au gives teachers access to many thousands of digital curriculum resources they can use to inform their own planning and support their teaching. The resources include learning objects, images, videos, audio, assessment resources, teacher resources and collections organised around common topics or themes. The resources are aligned to the endorsed areas of the *Australian Curriculum*.

Logging in: Teachers in Tasmanian Government schools log in using their DoE username and password.

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<p><b>McIntosh, A. (2014)</b></p> <p><b>Mental Computation: A Strategies Approach</b> (A series of six modules focusing on teaching mental computation strategies)</p>	<p>Modules 1-6 are available through Scootle. Log in and search Mental Computation.</p> <p>Logging in: Teachers in Tasmanian Government schools log in using their DoE username and password.</p>
<p><b>Australian Curriculum Lessons</b></p>	<p><i>Australian Curriculum Lessons</i> is an excellent user-submitted site that depends on teachers to post their great lessons so that other teachers can get ideas and lessons to use in the classroom.</p> <p><a href="http://www.australiancurriculumlessons.com.au/category/english-lessons/">http://www.australiancurriculumlessons.com.au/category/english-lessons/</a></p>
<p><b>NRICH</b></p>	<p>Problem-solving tasks and ideas from the University of Cambridge.</p> <p><a href="http://nrich.maths.org/frontpage">http://nrich.maths.org/frontpage</a></p>
<p> <b>Teachertube</b></p>	<p>A collection of videos, audios, photos, blogs and documents for teachers, parents and students.</p> <p><a href="http://www.teachertube.com">www.teachertube.com</a></p>
<p><b>Top Drawer Teachers</b></p>	<p>Top Drawer Teachers has a wealth of ideas on key mathematical ideas including advice on assessment, planning and student tasks. Developed by the Australian Association of Mathematics Teachers.</p> <p><a href="http://topdrawer.aamt.edu.au/">http://topdrawer.aamt.edu.au/</a></p>
<p><b>NSW Numeracy Continuum</b></p>	<p>NSW Numeracy Continuum – support in teaching key mathematical ideas.</p> <p><a href="http://www.numeracycontinuum.com/">http://www.numeracycontinuum.com/</a></p>
<p><b>NZ Maths</b></p>	<p>New Zealand Maths site – lots of resources and ideas to support teaching mathematics for numeracy</p> <p><a href="http://nzmaths.co.nz/">http://nzmaths.co.nz/</a></p>
<p><b>Queensland Curriculum and Assessment Authority</b></p>	<p>Support for planning and teaching Australian Curriculum Mathematics</p> <p><a href="https://www.qcaa.qld.edu.au/p-10/aciq/p-10-mathematics">https://www.qcaa.qld.edu.au/p-10/aciq/p-10-mathematics</a></p>
<p><b>Effective Numeracy Teaching 7–10 ( Victorian Department of Education)</b></p>	<p>A comprehensive discussion of effective numeracy practices in Years 7–10</p> <p><a href="http://www.agtv.vic.edu.au/files/Website%202015/keycharnumeracy7–10.pdf">http://www.agtv.vic.edu.au/files/Website%202015/keycharnumeracy7–10.pdf</a></p>
<p><b>Victorian Mathematics Developmental Continuum</b></p>	<p>Support for key ideas in mathematics</p> <p><a href="http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/continuum/Pages/mathcontin.aspx">http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/continuum/Pages/mathcontin.aspx</a></p>



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